

Paper Reference(s)

**6666/01**

**Edexcel GCE  
Core Mathematics C4  
Bronze Level B3**

**Time: 1 hour 30 minutes**

**Materials required for examination papers**

Mathematical Formulae (Green)

**Items included with question**

Nil

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

**Instructions to Candidates**

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Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

**Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper. The total mark for this paper is 75.

**Advice to Candidates**

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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**Suggested grade boundaries for this paper:**

<b>A*</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>69</b>	<b>61</b>	<b>54</b>	<b>49</b>	<b>43</b>	<b>37</b>

1. A curve  $C$  has the equation  $y^2 - 3y = x^3 + 8$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

(4)

(b) Hence find the gradient of  $C$  at the point where  $y = 3$ .

(3)

January 2009

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2. (a) Use integration by parts to find  $\int xe^x dx$ .

(3)

(b) Hence find  $\int x^2e^x dx$ .

(3)

June 2008

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3. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5},$$

in ascending powers of  $x$ , up to and including the term in  $x^2$ , giving each term as a simplified fraction.

(5)

Given that the binomial expansion of  $\frac{2+kx}{(2-5x)^2}$ ,  $|x| < \frac{2}{5}$ , is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots,$$

(b) find the value of the constant  $k$ ,

(2)

(c) find the value of the constant  $A$ .

(2)

January 2012

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4. A curve has equation  $3x^2 - y^2 + xy = 4$ . The points  $P$  and  $Q$  lie on the curve. The gradient of the tangent to the curve is  $\frac{8}{3}$  at  $P$  and at  $Q$ .

(a) Use implicit differentiation to show that  $y - 2x = 0$  at  $P$  and at  $Q$ .

(6)

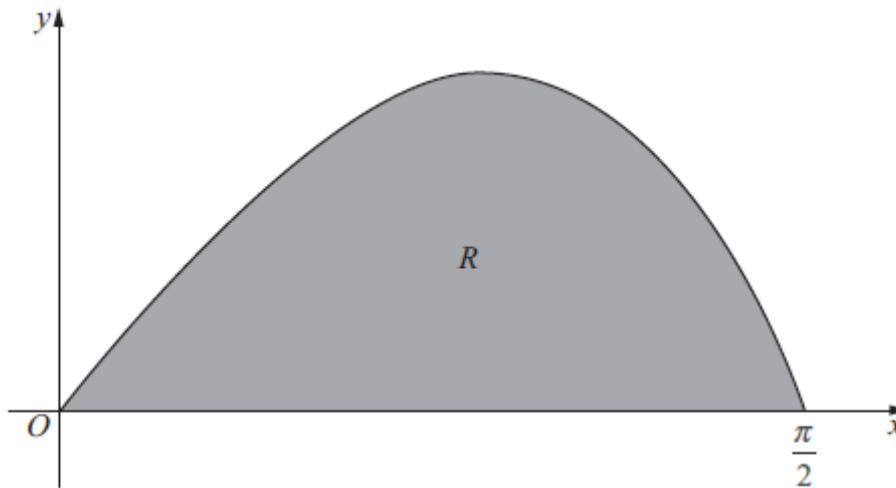
(b) Find the coordinates of  $P$  and  $Q$ .

(3)

**June 2008**

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5.



**Figure 3**

Figure 3 shows a sketch of the curve with equation  $y = \frac{2 \sin 2x}{1 + \cos x}$ ,  $0 \leq x \leq \frac{\pi}{2}$ .

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve and the  $x$ -axis.

The table below shows corresponding values of  $x$  and  $y$  for  $y = \frac{2 \sin 2x}{1 + \cos x}$ .

$x$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$y$	0		1.17157	1.02280	0

- (a) Complete the table above giving the missing value of  $y$  to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of  $y$  in the completed table, to obtain an estimate for the area of  $R$ , giving your answer to 4 decimal places. (3)
- (c) Using the substitution  $u = 1 + \cos x$ , or otherwise, show that

$$\int \frac{2 \sin 2x}{1 + \cos x} dx = 4 \ln(1 + \cos x) - 4 \cos x + k,$$

where  $k$  is a constant.

(5)

- (d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

**January 2012**

6. Relative to a fixed origin  $O$ , the point  $A$  has position vector  $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$  and the point  $B$  has position vector  $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$ .

The line  $l$  has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where  $a$ ,  $b$  and  $c$  are constants and  $\lambda$  is a parameter.

Given that the point  $A$  lies on the line  $l$ ,

- (a) find the value of  $a$ . (3)

Given also that the vector  $\overrightarrow{AB}$  is perpendicular to  $l$ ,

- (b) find the values of  $b$  and  $c$ , (5)

- (c) find the distance  $AB$ . (2)

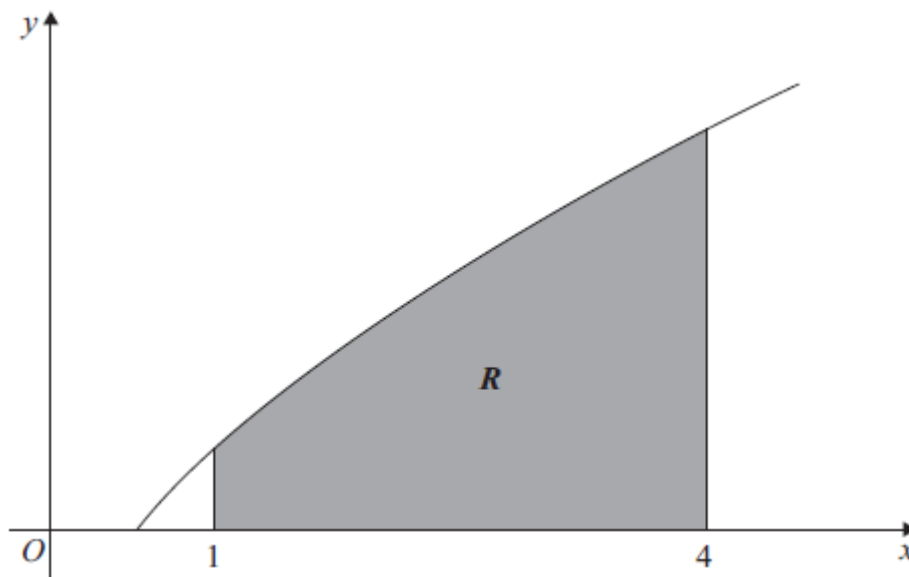
The image of the point  $B$  after reflection in the line  $l$  is the point  $B'$ .

- (d) Find the position vector of the point  $B'$ . (2)

**June 2013 (R)**

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7.



**Figure 3**

Figure 3 shows a sketch of part of the curve with equation  $y = x^{\frac{1}{2}} \ln 2x$ .

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve, the  $x$ -axis and the lines  $x = 1$  and  $x = 4$ .

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of  $R$ , giving your answer to 2 decimal places.

**(4)**

(b) Find  $\int x^{\frac{1}{2}} \ln 2x \, dx$ .

**(4)**

(c) Hence find the exact area of  $R$ , giving your answer in the form  $a \ln 2 + b$ , where  $a$  and  $b$  are exact constants.

**(3)**

**June 2012**

8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at  $3^{\circ}\text{C}$  and  $t$  minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is  $\theta^{\circ}\text{C}$ .

The rate of change of the temperature of the water in the bottle is modelled by the differential equation

$$\frac{d\theta}{dt} = \frac{(3 - \theta)}{125}.$$

- (a) By solving the differential equation, show that

$$\theta = Ae^{-0.008t} + 3,$$

where  $A$  is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was  $16^{\circ}\text{C}$ ,

- (b) find the time taken for the temperature of the water in the bottle to fall to  $10^{\circ}\text{C}$ , giving your answer to the nearest minute.

(5)

January 2013

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**TOTAL FOR PAPER: 75 MARKS**

**END**

Question Number	Scheme	Marks
1 (a)	<p>C: <math>y^2 - 3y = x^3 + 8</math></p> <p><del><math>\left\{ \frac{dy}{dx} \right\}</math></del> <math>2y \frac{dy}{dx} - 3 \frac{dy}{dx} = 3x^2</math></p> <p><math>(2y-3) \frac{dy}{dx} = 3x^2</math></p> <p><math>\frac{dy}{dx} = \frac{3x^2}{2y-3}</math></p>	<p>Differentiates implicitly to include either <math>\pm ky \frac{dy}{dx}</math> or <math>\pm 3 \frac{dy}{dx}</math>. (Ignore <math>\left( \frac{dy}{dx} = \right)</math>.) M1</p> <p>Correct equation. A1</p> <p>A correct (condoning sign error) attempt to combine or factorise their '<math>2y \frac{dy}{dx} - 3 \frac{dy}{dx}</math>'. M1</p> <p>Can be implied. <math>\frac{3x^2}{2y-3}</math> A1 oe</p>
(b)	<p><math>y = 3 \Rightarrow 9 - 3(3) = x^3 + 8</math></p> <p><math>x^3 = -8 \Rightarrow \underline{x = -2}</math></p> <p><math>(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4</math></p>	<p>Substitutes <math>y = 3</math> into C. M1</p> <p>Only <math>\underline{x = -2}</math> A1</p> <p><math>\frac{dy}{dx} = 4</math> from correct working.</p> <p>Also can be ft using their '<math>x</math>' value and <math>y = 3</math> in the correct part (a) of <math>\frac{dy}{dx} = \frac{3x^2}{2y-3}</math> A1 <math>\sqrt{}</math></p> <p>(3) [7]</p>

Question Number	Scheme	Marks
2. (a)	<p><math>\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}</math></p> <p><math>\int x e^x dx = x e^x - \int e^x \cdot 1 dx</math></p> <p><math>= x e^x - \int e^x dx</math></p> <p><math>= x e^x - e^x (+ c)</math></p>	<p>M1 A1</p> <p>A1 (3)</p>
(b)	<p><math>\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}</math></p> <p><math>\int x^2 e^x dx = x^2 e^x - \int e^x \cdot 2x dx</math></p> <p><math>= x^2 e^x - 2 \int x e^x dx</math></p> <p><math>= x^2 e^x - 2(x e^x - e^x) + c</math></p>	<p>M1 A1</p> <p>A1 (3)</p> <p>(6 marks)</p>



Question Number	Scheme	Marks
3. (a)	$\frac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}} \left(1 - \frac{5x}{2}\right)^{-2} = \frac{1}{4} \left(1 - \frac{5x}{2}\right)^{-2}$ $= \left\{ \frac{1}{4} \right\} \left[ 1 + (-2)(**x) + \frac{(-2)(-3)}{2!} (**x)^2 + \dots \right]$ $= \left\{ \frac{1}{4} \right\} \left[ 1 + (-2) \left( -\frac{5x}{2} \right) + \frac{(-2)(-3)}{2!} \left( -\frac{5x}{2} \right)^2 + \dots \right]$ $= \frac{1}{4} \left[ 1 + 5x; + \frac{75}{4} x^2 + \dots \right]$ $= \frac{1}{4} + \frac{5}{4} x; + \frac{75}{16} x^2 + \dots$	<u>B1</u> M1 A1ft  A1; A1 <b>[5]</b>
(b)	$\left\{ \frac{2+kx}{(2-5x)^2} \right\} = (2+kx) \left( \frac{1}{4} + \frac{5}{4}x + \left\{ \frac{75}{16}x^2 + \dots \right\} \right)$ <p><i>Can be implied by later work even in part (c).</i></p> <p><i>x terms:</i> <math>\frac{2(5x)}{4} + \frac{kx}{4} = \frac{7x}{4}</math></p> <p>giving, <math>10 + k = 7 \Rightarrow \underline{k = -3}</math></p>	M1  A1 <b>[2]</b>
(c)	<p><i>x<sup>2</sup> terms:</i> <math>\frac{150x^2}{16} + \frac{5kx^2}{4}</math></p> <p>So, <math>A = \frac{75}{8} + \frac{5(-3)}{4} = \frac{75}{8} - \frac{15}{4} = \underline{\frac{45}{8}}</math></p>	M1 A1 <u><math>\frac{45}{8}</math></u> or <u><math>5\frac{5}{8}</math></u> or <u>5.625</u> <b>[2]</b> <b>(9 marks)</b>

4. (a)	$3x^2 - y^2 + xy = 4 \quad (\text{eqn } *)$ $\left\{ \begin{array}{l} \cancel{6x} \\ \cancel{2y} \end{array} \right\} \times \left\{ \frac{dy}{dx} \right\} + \left( y + x \frac{dy}{dx} \right) = \underline{0}$ $\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$ <p>giving <math>-18x - 3y = 8x - 16y</math></p> <p>giving <math>13y = 26x</math></p> <p>Hence, <math>y = 2x \Rightarrow \underline{y - 2x = 0}</math></p>	M1 B1 A1  M1  M1  A1 cso (6)
(b)	<p>At P &amp; Q, <math>y = 2x</math>. Substituting into eqn *</p> <p>gives <math>3x^2 - (2x)^2 + x(2x) = 4</math></p> <p>Simplifying gives, <math>x^2 = 4 \Rightarrow \underline{x = \pm 2}</math></p> <p><math>y = 2x \Rightarrow y = \pm 4</math>, hence coordinates are <u>(2,4)</u> and <u>(-2,-4)</u></p>	M1  A1  A1 (3) <b>(9 marks)</b>

Question Number	Scheme	Marks
5. (a)	0.73508	B1 <b>cao</b> (1)
(b)	$\text{Area} \approx \frac{1}{2} \times \frac{\pi}{8} \times [0 + 2(\text{their } 0.73508 + 1.17157 + 1.02280) + 0]$ $= \frac{\pi}{16} \times 5.8589... = 1.150392325... = 1.1504 \text{ (4 dp)}$	B1 <u>M1</u> A1 (3) awrt 1.1504
(c)	$\{u = 1 + \cos x\} \Rightarrow \frac{du}{dx} = -\sin x$ $\left\{ \int \frac{2 \sin 2x}{(1 + \cos x)} dx = \int \frac{2(2 \sin x \cos x)}{(1 + \cos x)} dx \right.$ $= \int \frac{4(u-1)}{u} \cdot (-1) du \left\{ = 4 \int \frac{(1-u)}{u} du \right\}$ $= 4 \int \left( \frac{1}{u} - 1 \right) du = 4(\ln u - u) + c$ $= 4 \ln(1 + \cos x) - 4(1 + \cos x) + c = 4 \ln(1 + \cos x) - 4 \cos x + k$	B1 $\sin 2x = 2 \sin x \cos x$ B1 M1 dM1 A1 cso (5)
(d)	<p><b>AG</b></p> $= \left[ 4 \ln \left( 1 + \cos \frac{\pi}{2} \right) - 4 \cos \frac{\pi}{2} \right] - \left[ 4 \ln(1 + \cos 0) - 4 \cos 0 \right]$ $= [4 \ln 1 - 0] - [4 \ln 2 - 4]$ $= 4 - 4 \ln 2 \{ = 1.227411278... \}$ $\text{Error} =  (4 - 4 \ln 2) - 1.1504... $ $= 0.0770112776... = 0.077 \text{ (2sf)}$	$\text{Applying limits } x = \frac{\pi}{2}$ $\text{and } x = 0 \text{ either way round.}$ M1 A1 $\pm 4(1 - \ln 2) \text{ or}$ $\pm(4 - 4 \ln 2) \text{ or awrt } \pm 1.2$ $\text{awrt } \pm 0.077$ $\text{or awrt } \pm 6.3(\%)$ A1 cso (3)
		<b>(12 marks)</b>

Question Number	Scheme	Marks
<p><b>6.</b></p> <p><b>(a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p> <p><b>(d)</b></p>	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}, \quad \overline{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}, \quad \overline{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$ $A \text{ is on } l, \text{ so } \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ $\{\mathbf{k}: 10 - \lambda = 6 \Rightarrow\} \lambda = 4$ $\{\mathbf{i}: a + 6\lambda = 21 \Rightarrow\} a + 6(4) = 21$ $a = -3$ $\left\{ \begin{array}{l} \overline{AB} \\ \overline{BA} \end{array} \right\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} \quad \left  \quad \left\{ \begin{array}{l} \overline{BA} \\ \overline{AB} \end{array} \right\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} \right.$ $\left\{ \overline{AB} \right\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \quad \left  \quad \left\{ \overline{BA} \right\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix} \right.$ $\{\overline{AB} \perp l \Rightarrow \overline{AB} \cdot \mathbf{d} = 0\} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4$ $\{\mathbf{j}: b + c\lambda = -17 \Rightarrow\} b + (-4)(4) = -17; \Rightarrow b = -1$ $ AB  = \sqrt{4^2 + 3^2 + 12^2} \quad \text{or} \quad  AB  = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}$ <p>So, <math> AB  = 13</math></p> $\overline{OB'} \{ = \overline{OA} + \overline{BA} \} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix} = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	<p><math>\lambda = 4</math> B1</p> <p>Substitutes their value of <math>\lambda</math> into <math>a + 6\lambda = 21</math> M1</p> <p><math>a = -3</math> A1</p> <p><b>cao</b> (3)</p> <p>Finds the difference between <math>\overline{OA}</math> and <math>\overline{OB}</math>. M1</p> <p>Ignore labelling.</p> <p>M1; A1 ft</p> <p>ddM1; A1 <b>cao</b> (5)</p> <p>M1</p> <p>A1 <b>cao</b> (2)</p> <p>M1; A1 <b>cao</b> (2)</p> <p>[12]</p>

Question Number	Scheme	Marks															
7. (a)	<table border="1" data-bbox="352 320 1165 445"> <thead> <tr> <th><math>x</math></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> </tr> </thead> <tbody> <tr> <td><math>y</math></td> <td><math>\ln 2</math></td> <td><math>\sqrt{2} \ln 4</math></td> <td><math>\sqrt{3} \ln 6</math></td> <td><math>2 \ln 8</math></td> </tr> <tr> <td></td> <td>0.6931</td> <td>1.9605</td> <td>3.1034</td> <td>4.1589</td> </tr> </tbody> </table>	$x$	1	2	3	4	$y$	$\ln 2$	$\sqrt{2} \ln 4$	$\sqrt{3} \ln 6$	$2 \ln 8$		0.6931	1.9605	3.1034	4.1589	M1
	$x$	1	2	3	4												
	$y$	$\ln 2$	$\sqrt{2} \ln 4$	$\sqrt{3} \ln 6$	$2 \ln 8$												
	0.6931	1.9605	3.1034	4.1589													
$\text{Area} = \frac{1}{2} \times 1( \dots )$ $\approx \dots (0.6931 + 2(1.9605 + 3.1034) + 4.1589)$ $\approx \frac{1}{2} \times 14.97989 \dots \approx 7.49$	B1 M1 A1 <b>(4)</b>																
(b)	$\int x^{\frac{1}{2}} \ln 2x \, dx = \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3} x^{\frac{3}{2}} \times \frac{1}{x} \, dx$ $= \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3} x^{\frac{1}{2}} \, dx$ $= \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \frac{4}{9} x^{\frac{3}{2}} \quad (+C)$	M1 A1  M1 A1 <b>(4)</b>															
(c)	$\left[ \frac{2}{3} x^{\frac{3}{2}} \ln 2x - \frac{4}{9} x^{\frac{3}{2}} \right]_1^4 = \left( \frac{2}{3} 4^{\frac{3}{2}} \ln 8 - \frac{4}{9} 4^{\frac{3}{2}} \right) - \left( \frac{2}{3} \ln 2 - \frac{4}{9} \right)$ $= (16 \ln 2 - \dots) - \dots \quad \text{Using or implying } \ln 2^n = n \ln 2$ $= \frac{46}{3} \ln 2 - \frac{28}{9}$	M1 M1 A1 <b>(3)</b>  <b>[11]</b>															

Question Number	Scheme	Marks
8. (a)	$\left\{ \frac{d\theta}{dt} = \frac{(3-\theta)}{125} \right\} \Rightarrow \int \frac{1}{3-\theta} d\theta = \int \frac{1}{125} dt \quad \text{or}$ $\int \frac{125}{3-\theta} d\theta = \int dt$ $-\ln(\theta - 3) = \frac{1}{125}t \{+ c\} \quad \text{or}$ $-\ln(3 - \theta) = \frac{1}{125}t \{+ c\}$ $\ln(\theta - 3) = -\frac{1}{125}t + c$ $\theta - 3 = e^{-\frac{1}{125}t + c} \quad \text{or} \quad e^{-\frac{1}{125}t} e^c$ $\theta = Ae^{-0.008t} + 3 \quad *$	<p>B1</p> <p>See notes.</p> <p>M1 A1</p> <p><b>Correct</b> completion to <math>\theta = Ae^{-0.008t} + 3</math>.</p> <p>A1</p> <p>[4]</p>
(b)	$\{t=0, \theta=16 \Rightarrow\} \quad 16 = Ae^{-0.008(0)} + 3;$ $\Rightarrow \underline{A=13}$ $10 = 13e^{-0.008t} + 3$ $e^{-0.008t} = \frac{7}{13} \Rightarrow -0.008t = \ln\left(\frac{7}{13}\right)$ $\left\{ t = \frac{\ln\left(\frac{7}{13}\right)}{(-0.008)} \right\} = 77.3799... = 77 \text{ (nearest minute)}$	<p>See notes.</p> <p>Substitutes <math>\theta = 10</math> into an equation of the form <math>\theta = Ae^{-0.008t} + 3</math>, or equivalent.</p> <p>Correct algebra to <math>-0.008t = \ln k</math>, where <math>k</math> is a positive value.</p> <p>M1; A1</p> <p>M1</p> <p>M1</p> <p>awrt 77 A1</p> <p>[5] 9</p>

## Question 1

A significant majority of candidates were able to score full marks on this question. In part (a), many candidates were able to differentiate implicitly and examiners noticed fewer candidates differentiating 8 incorrectly with respect to  $x$  to give 8. In part (b), many candidates were able to substitute  $y = 3$  into  $C$  leading to the correct  $x$ -coordinate of  $-2$ . Several candidates either rearranged their  $C$  equation incorrectly to give  $x = 2$  or had difficulty finding the cube root of  $-8$ . Some weaker candidates did not substitute  $y = 3$  into  $C$ , but substituted  $y = 3$  into the  $\frac{dy}{dx}$  expression to give a gradient of  $x^2$ .

## Question 2

In part (a), many candidates were able to use integration by parts in the right direction to produce a correct solution. Common errors included integrating  $e^x$  incorrectly to give  $\ln x$  or applying the by parts formula in the wrong direction by assigning  $u$  as  $e^x$  to be differentiated and  $\frac{dv}{dx}$  as  $x$  to be integrated.

Many candidates were able to make a good start to part (b), by assigning  $u$  as  $x^2$  and  $\frac{dv}{dx}$  as  $e^x$  and again correctly applying the integration by parts formula. At this point, when faced with integrating  $2xe^x$ , some candidates did not make the connection with their answer to part (a) and made little progress, whilst others independently applied the by parts formula again. A significant proportion of candidates made a bracketing error and usually gave an incorrect answer of  $e^x(x^2 - 2x - 2) + c$ .

In part (b), a few candidates proceeded by assigning  $u$  as  $x$  and  $\frac{dv}{dx}$  as  $xe^x$  and then used their answer to part (a) to obtain  $v$ . These candidates were usually produced a correct solution.

## Question 3

This question was also generally well answered with about 50% of candidates obtaining all of the 9 marks available.

In part (a), a minority of candidates were unable to carry out the first step of writing  $\frac{1}{(2-5x)^2}$  as  $\frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$ , with the  $\frac{1}{4}$  outside the brackets usually written incorrectly as either 1 or  $\frac{1}{2}$ . Many candidates were able to use a correct method for expanding a binomial expression of the form  $(1+ax)^n$ . A variety of incorrect values of  $a$  were seen, with the most common being either  $\frac{5}{2}$ , 5 or  $-5$ . Some candidates, having correctly expanded  $\left(1-\frac{5x}{2}\right)^{-2}$ , forgot to multiply their expansion by  $\frac{1}{4}$ . As expected, sign errors, bracketing errors, and simplification errors were also seen in this part.

In parts (b) and (c), most candidates realised that they needed to multiply  $(2+kx)$  by their binomial expansion from part (a) and equate their  $x$  and  $x^2$  coefficients in order to find both  $k$  and  $A$ . A small minority, however, attempted to divide  $(2+kx)$  by their part (a) expansion.

Other candidates omitted the brackets around  $2+kx$ , although they progressed as if these “invisible” brackets were really there.

In part (b), a significant minority of candidates used an incorrect method of multiplying  $(2+kx)$  by the first term (usually  $\frac{1}{4}$ ) of their binomial expansion, and equating the result to  $\frac{1}{2}$  in order to find  $k$ . In part (c), these candidates also multiplied  $(2+kx)$  by the third term (usually  $\frac{75}{16}x^2$ ) of their binomial expansion and equated this to  $Ax^2$  in order to find  $A$ .

A few candidates in parts (b) and (c) applied an alternative method of multiplying out  $(2-5x)^2\left(\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots\right)$  and equating the result to  $(2+kx)$ , in order to correctly find both  $k$  and  $A$ .

#### Question 4

This question was generally well done with a majority of candidates scoring at least 6 of the 9 marks available.

In part (a), implicit differentiation was well handled with most candidates appreciating the need to apply the product rule to the  $xy$  term. A few candidates failed to differentiate the constant term and some wrote " $\frac{dy}{dx} = \dots$ " before starting to differentiate the equation. After differentiating implicitly, the majority of candidates rearranged the resulting equation to make  $\frac{dy}{dx}$  the subject before substituting  $\frac{dy}{dx}$  as  $\frac{8}{3}$  rather than substituting  $\frac{8}{3}$  for  $\frac{dy}{dx}$  in their differentiated equation. Many candidates were able to prove the result of  $y-2x=0$ . A surprising number of candidates when faced with manipulating the equation  $\frac{6x+y}{2y-x} = \frac{8}{3}$ , separated the fraction to incorrectly form two equations  $6x+y=8$  &  $2y-x=3$  and then proceeded to solve these equations simultaneously.

Some candidates, who were unsuccessful in completing part (a), gave up on the whole question even though it was still possible for them to obtain full marks in part (b). Other candidates, however, did not realise that they were expected to substitute  $y=2x$  into the equation of the curve and made no creditable progress with this part. Those candidates who used the substitution  $y=2x$  made fewer errors than those who used the substitution  $x=\frac{y}{2}$ . The most common errors in this part were for candidates to rewrite  $-y^2$  as either  $4x^2$  or  $-2x^2$ ; or to solve the equation  $x^2=4$  to give only  $x=2$  or even  $x=\pm 4$ . On finding  $x=\pm 2$ , some candidates went onto substitute these values back into the equation of the curve, forming a quadratic equation and usually finding “extra” unwanted points rather than simply doubling their two values of  $x$  to find the corresponding two values for  $y$ . Most candidates who progressed this far were able to link their values of  $x$  and  $y$  together, usually as coordinates.

#### Question 5

In part (a), virtually all candidates were able to find the  $y$ -value corresponding to  $x = \frac{\pi}{8}$ .

In part (b), most candidates were able to apply the trapezium rule correctly to find the correct estimate with the most common errors being candidates writing  $h$  as either  $\frac{\pi}{10}$ ,  $\frac{\pi}{4}$  or  $\frac{\pi}{16}$ ; or

candidates rounding incorrectly to give 1.1503. Few bracketing errors in part (b) were encountered in this session.

Part (c) provided a diverse range of solutions. Most candidates followed the advice given in the question to use the substitution of  $u = 1 + \cos x$ , so obtaining  $\frac{du}{dx} = -\sin x$  (or occasionally  $\sin x$ ), as well as using the double angle formula for sine to process the numerator of the integral. Whilst some students found the conversion of the given integral to an expression in  $u$  beyond them, many more were able to reach an integral of the form  $k \int \frac{(u-1)}{u} (du)$ . Whilst most candidates reaching this stage then correctly divided through by  $u$  and integrated term by term to reach an expression of the form  $k(\ln u - u)$ , a few resorted to integration by parts and were generally less successful. A significant proportion of candidates lost the final accuracy mark as a result of not showing how their constant of integration could be combined with the  $-4$  from their integration to give the stated  $k$  in the question; some found a value for  $k$  (usually 4) or some simply failed to state the final result.

In part (d), those candidates who were unable to complete part (c) often realised that they were still able to attempt part (d). The use of limits for either  $x$  or for  $u$  was generally successfully completed to obtain the value 1.227 or  $4 - 4 \ln 2$ , but the final step of finding the error was not so successfully tackled.

## Question 6

This was a well answered question on vectors with about 56% of candidates gaining at least 10 of the 12 marks available and about 23% of candidates gaining all 12 marks. Part (d) required the more able candidates to think for themselves.

In part (a), most candidates set the line  $l$  equal to the point A and equated the  $k$  components to find the value of  $\lambda$ . This was followed by equating the  $i$  components to find the correct value of  $a$ , although some candidates, however, incorrectly found  $a = 3$  from  $a + 6(4) = 21$ . A small minority of candidates wrote down equations for the  $i$  and  $k$  components and solved these simultaneously to find  $a$ .

In part (b), almost all candidates found  $\overrightarrow{AB}$  and many applied  $\overrightarrow{AB} \cdot \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 0$  in order to find

the value of  $c$  and proceeded to find the value of  $b$ . Some candidates incorrectly applied

$\begin{pmatrix} -3 + 6\lambda \\ b + c\lambda \\ 10 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 0$  and thus made little progress. Some candidates incorrectly found  $c = 4$

or 3 from a correct  $24 + 3c - 12 = 0$  whilst others incorrectly found  $b = -33, 33$  or 1 from a correct  $b = (-4)(4) = -17$ .

In part (c), most candidates were able to find the correct distance  $AB$  and few errors were seen.

In part (d), the majority of candidates were not able to use the information given earlier in the question and many of them left this part blank. The most common error of those who attempted this part was to write down  $B'$  as  $-25\mathbf{i} + 14\mathbf{j} - 18\mathbf{k}$ . Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. Some



candidates, by deducing that  $A$  was the midpoint of  $B$  and  $B'$  were able to write down  $\frac{x+25}{2} = 21$ ,  $\frac{y-14}{2} = -17$  and  $\frac{z+18}{2} = 6$  in order to find the position vector of  $B'$ .

### Question 7

Part (a) was accessible to most candidates. This was the first time in recent years that candidates had to produce their own table. This, in general, they did well although the number of decimal places recorded often seemed too few to be working towards a final accuracy of 2 decimal places. To obtain an answer of this accuracy, you should tabulate figures to at least 3 decimal places. Of course, the examiners have no means of knowing what figures the candidate has in their calculator and as long as there was some tabulation, or the exact expressions  $\ln 2$ ,  $\sqrt{2} \ln 4$ ,  $\sqrt{3} \ln 6$  and  $2 \ln 8$  were given, and the working showed that the correct formula was known, then, if 7.49 was given as the answer, the candidate was given the benefit of the doubt. A few candidates confused the number of strips with the number of ordinates but these were fewer than in some recent examinations.

In part (b), most knew that they had to use integration by parts and most attempted this in the “right direction” attempting to integrate  $x^{\frac{1}{2}}$ , which was usually correct, and differentiate  $\ln 2x$ , for which the incorrect  $\frac{1}{2x}$  was often seen. Many who reached the intermediate stage correctly had difficulty with  $\int \frac{2}{3}x^{\frac{3}{2}} \cdot \frac{1}{x} dx$ , failing to divide  $x^{\frac{3}{2}}$  by  $x$ . Fully correct solutions to part (b) were not common.

If they had an answer to part (b) in the correct form, then most candidates showed that they could complete the question by using the limits correctly and then using the power rule for logs to obtain an answer in the form specified. There were many errors of detail in candidates’ solutions to this question but more than 50% of candidates gained eight or more of the available eleven marks..

### Question 8

Some candidates did not attempt to separate the variables in Q8(a). They were also not able to deal with the context of the question in Q8(b).

In Q8(a), those candidates who were able to separate the variables, were usually able to integrate both sides correctly, although a number made a sign error by integrating  $\frac{1}{3-\theta}$  to obtain  $\ln|3-\theta|$ . A significant number of candidates omitted the constant of integration “+ c” and so were not able to gain the final mark. A significant number of candidates did not show sufficient steps in order to progress from  $-\ln|3-\theta| = \frac{1}{125}t + c$  to the result  $\theta = Ae^{-0.008t} + 3$ . Common errors included candidates removing their logarithms incorrectly to give an equation of the form  $3-\theta = e^{-\frac{1}{125}t} + A$  or candidates stating the constant  $A$  as  $-1$ .

Q8(b) was often better answered with some candidates scoring no marks in q8(a) and full marks in Q8(b). Those candidates who used  $\theta = Ae^{-0.008t} + 3$  were more successful in this part. They were usually able to write down the condition  $\theta = 16$  when  $t = 0$  in order to find  $A = 13$ . Some candidates misinterpreted the context of the question to write down the condition  $\theta = 6$  when  $t = 0$ , yielding the result of  $A = 3$ . Other incorrect values of  $A$  seen by examiners included  $-1$ ,  $16$  or  $1$ . Many candidates who found  $A$  correctly were usually able to substitute  $\theta = 10$  into  $\theta = 16e^{-0.008t} + 3$  and manipulate the result correctly in order to find the correct time.

### Statistics for C4 Practice Paper Bronze Level B3

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	7		86	6.02		6.76	6.35	5.50	4.60	3.52	2.07
2	6		76	4.53		5.40	4.87	4.24	3.38	2.32	1.11
3	9		78	7.00	8.87	8.08	6.89	5.87	4.48	4.04	1.83
4	9		71	6.43		8.15	6.86	5.52	4.01	2.59	1.18
5	12		70	8.34	11.49	10.12	7.73	5.92	4.44	3.52	1.60
6	12		69	8.24	11.17	9.31	7.40	4.92	3.69	2.70	0.96
7	11		67	7.32	10.54	9.14	7.50	5.85	4.39	3.04	1.60
8	9	0	54	4.85	8.15	6.01	3.95	2.55	1.60	0.99	0.34
	<b>75</b>		<b>70</b>	<b>52.73</b>		<b>62.97</b>	<b>51.55</b>	<b>40.37</b>	<b>30.59</b>	<b>22.72</b>	<b>10.69</b>