Paper Reference(s)

# 6666/01

# Edexcel GCE

## Core Mathematics C4
## Gold Level (Hardest) G4

## Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**Suggested grade boundaries for this paper:**

| A* | A | B | C | D | E |
|---|---|---|---|---|---|
| 62 | 52 | 42 | 36 | 30 | 26 |

**1.** (*a*) Find $\int x^2 e^x \mathrm{d}x$ .

**(5)**

(*b*) Hence find the exact value of $\int_0^1 x^2 e^x \mathrm{d}x$ .

**(2)**

**June 2013**

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**2.** Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x + 1)^2} \, \mathrm{d}x.$$

**(6)**

**June 2007**

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**3.**

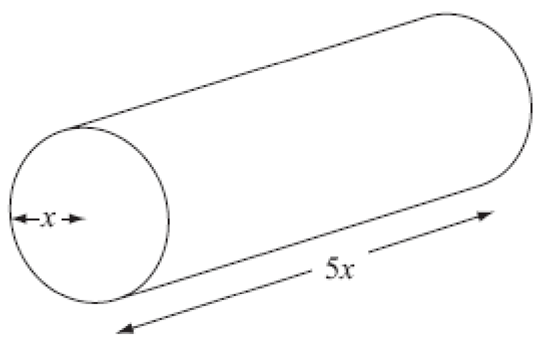


**Figure 2**

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After $t$ seconds the radius of the rod is $x$ cm and the length of the rod is $5x$ cm.

The cross-sectional area of the rod is increasing at the constant rate of 0.032 $\text{cm}^2\,\text{s}^{-1}$.

(*a*) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

**(4)**

(*b*) Find the rate of increase of the volume of the rod when $x = 2$.

**(4)**

**June 2008**

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**4.** (i) Find $\int \ln\left(\frac{x}{2}\right) \, \mathrm{d}x$.

**(4)**

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \; \mathrm{d}x$.

**(5)**

**January 2008**

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**5.** (*a*) Find $\int \frac{9x+6}{x} \, \mathrm{d}x, \quad x > 0$.

**(2)**

(*b*) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = \mathrm{g}(x)$.

**(6)**

**January 2010**

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**6.**

$$\mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(*a*) Show that $\mathrm{f}(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

**(3)**

(*b*) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta \, \mathrm{f}(\theta) \; \mathrm{d}\theta$.

**(7)**

**June 2010**

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**7.** Relative to a fixed origin $O$, the point $A$ has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point $B$ has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point $C$ has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line $l$ passes through the points $A$ and $B$.

(*a*) Find a vector equation for the line $l$.

**(3)**

(*b*) Find $\left|\overrightarrow{CB}\right|$.

**(2)**

(*c*) Find the size of the acute angle between the line segment $CB$ and the line $l$, giving your answer in degrees to 1 decimal place.

**(3)**

(*d*) Find the shortest distance from the point $C$ to the line $l$.

**(3)**

The point $X$ lies on $l$. Given that the vector $\overrightarrow{CX}$ is perpendicular to $l$,

(*e*) find the area of the triangle $CXB$, giving your answer to 3 significant figures.

**(3)**

**June 2009**

**8.** Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $\text{cm}^3\text{s}^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 $\text{cm}^2$.

(*a*) Show that at time *t* seconds, the height *h* cm of liquid in the cylinder satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - k\sqrt{h},$$

where *k* is a positive constant.

**(3)**

When $h = 25$, water is leaking out of the hole at 400 $\text{cm}^3\text{s}^{-1}$.

(*b*) Show that $k = 0.02$.

**(1)**

(*c*) Separate the variables of the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0.4 - 0.02\sqrt{h}$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\,\mathrm{d}h\,.$$

**(2)**

Using the substitution $h = (20 - x)^2$, or otherwise,

(*d*) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\,\mathrm{d}h$.

**(6)**

(*e*) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

**(1)**

**January 2008**

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**TOTAL FOR PAPER: 75 MARKS**

**END**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 1. (a) | $\int x^2 e^x \, dx$, 1st Application: $\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$, 2nd Application: $\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$ | | |
| | $= x^2 e^x - \int 2xe^x dx$ | $x^2 e^x - \int \lambda x e^x \{dx\}, \ \lambda > 0$ | M1 |
| | | $x^2 e^x - \int 2xe^x \{dx\}$ | A1 oe |
| | $= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$ | **Either** $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$ | M1 |
| | $= x^2 e^x - 2(xe^x - e^x) \ \{+c\}$ | $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$ | M1 |
| | | Correct answer, with/without $+c$ | A1 |
| | | | **(5)** |
| (b) | $\left\{\left[x^2 e^x - 2(xe^x - e^x)\right]_0^1\right\}$<br>$= \left(1^2 e^1 - 2(1e^1 - e^1)\right) - \left(0^2 e^0 - 2(0e^0 - e^0)\right)$ | Applies limits of 1 and 0 to an expression of the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round. | M1 |
| | $= e - 2$ | $e - 2$ **cso** | A1 oe |
| | | | **(2)** |
| | | | **[7]** |

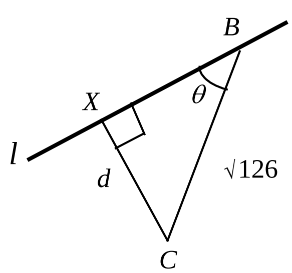
| Question Number | Scheme | | Marks |
|---|---|---|---|
| **2.** | $\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx$ , with substitution $u = 2^x$ | | |
| | $\dfrac{du}{dx} = 2^x.\ln 2 \;\Rightarrow\; \dfrac{dx}{du} = \dfrac{1}{2^x.\ln 2}$ | $\frac{du}{dx} = 2^x.\ln 2$ or $\frac{du}{dx} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx} = \ln 2$ | B1 |
| | $\displaystyle\int \frac{2^x}{(2^x+1)^2}\,dx = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$ | $\displaystyle k\int \frac{1}{(u+1)^2}\,du$ where $k$ is constant | M1 $*$ |
| | $= \left(\dfrac{1}{\ln 2}\right)\left(\dfrac{-1}{(u+1)}\right) + c$ | $(u+1)^{-2} \to a(u+1)^{-1}$<br>$(u+1)^{-2} \to -1.(u+1)^{-1}$ | M1<br>A1 |
| | change limits: when $x$ = 0 & $x$ = 1 then $u$ = 1 & $u$ = 2 | | |
| | $\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$ | | |
| | $= \dfrac{1}{\ln 2}\left[\left(-\dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right)\right]$ | Correct use of limits $u$ = 1 and $u$ = 2 | depM1 $*$ |
| | $= \dfrac{1}{6\ln 2}$ | $\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3n2}$<br>**Exact value only!** | A1 aef<br>**[6]** |
| | Alternatively candidate can revert back to $x$ … | | |
| | $\displaystyle\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$ | | |
| | $= \dfrac{1}{\ln 2}\left[\left(-\dfrac{1}{3}\right) - \left(-\dfrac{1}{2}\right)\right]$ | Correct use of limits $x$ = 0 and $x$ = 1 | depM1 $*$ |
| | $= \underline{\dfrac{1}{6\ln 2}}$ | $\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$<br>**Exact value only!** | A1 aef |
| | | | **6 marks** |

| | | | |
|---|---|---|---|
| **3.** | (*a*) | From question, $\frac{dA}{dt} = 0.032$ | B1 |
| | | $\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} = \right\} 2\pi x$ | B1 |
| | | $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}$; $\left\{= \frac{0.016}{\pi x}\right\}$ | M1 |
| | | When $x = 2\,\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$ | |
| | | Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s$^{-1}$) | A1 cso (4) |
| | (*b*) | $V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$ | B1 |
| | | $\frac{dV}{dx} = 15\pi x^2$ | B1 ft |
| | | $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2 . \left(\frac{0.016}{\pi x}\right)$; $\{= 0.24x\}$ | M1 |
| | | When $x = 2\,\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ (cm$^3$ s$^{-1}$) | A1 (4) |
| | | | **(8 marks)** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** (i) | $\int \ln\left(\frac{x}{2}\right)\,dx = \int 1.\ln\left(\frac{x}{2}\right)\,dx \Rightarrow \left\{\begin{matrix} u=\ln\left(\frac{x}{2}\right) & \Rightarrow & \frac{du}{dx}=\frac{\frac{1}{2}}{\frac{x}{2}}=\frac{1}{x} \\ \frac{dv}{dx}=1 & \Rightarrow & v=x \end{matrix}\right\}$ | | |
| | $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln\left(\frac{x}{2}\right)-\int x.\frac{1}{x}\,dx$ | Use of 'integration by parts' formula in the correct direction.<br>Correct expression. | M1<br>A1 |
| | $= x\ln\left(\frac{x}{2}\right)-\int \underline{1}\,dx$ | An attempt to multiply $x$ by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$. | <u>dM1</u> |
| | $= x\ln\left(\frac{x}{2}\right)-x+c$ | Correct integration with + $c$ | A1 aef<br>**[4]** |
| (ii) | $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$ | | |
| | $\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \tfrac{1}{2}(\pm 1 \pm \cos 2x)}\right]$ | Consideration of double angle formula for $\cos 2x$ | M1 |
| | $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1-\cos 2x})\,dx$ | | |
| | $= \frac{1}{2}\left[\underline{x-\tfrac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ | <u>Integrating to give</u> <u>$\pm ax \pm b\sin 2x$</u>; $a, b \neq 0$<br>Correct result of anything equivalent to $\frac{1}{2}x-\frac{1}{4}\sin 2x$ | dM1<br>A1 |
| | $= \frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin(\pi)}{2}\right)-\left(\frac{\pi}{4}-\frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)\right]$<br>$= \frac{1}{2}\left[\left(\frac{\pi}{2}-0\right)-\left(\frac{\pi}{4}-\frac{1}{2}\right)\right]$ | Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round. | ddM1 |
| | $= \frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right) = \frac{\pi}{8}+\frac{1}{4}$ | $\underline{\tfrac{1}{2}\left(\tfrac{\pi}{4}+\tfrac{1}{2}\right)}$ or $\underline{\tfrac{\pi}{8}+\tfrac{1}{4}}$ or $\underline{\tfrac{\pi}{8}+\tfrac{2}{8}}$<br>Candidate must collect their $\pi$ term and constant term together for A1<br>No fluked answers, hence **cso**. | A1 **aef**, **cso**<br>**[5]** |
| | | | **9 marks** |

| Question Number | Scheme | Marks |
|---|---|---|
| Q5 | (a) $\int \frac{9x+6}{x}\,dx = \int\left(9+\frac{6}{x}\right)dx$ | M1 |
| | $= 9x + 6\ln x \ (+C)$ | A1 (2) |
| | (b) $\int \frac{1}{y^{\frac{1}{3}}}\,dy = \int \frac{9x+6}{x}\,dx$ Integral signs not necessary | B1 |
| | $\int y^{-\frac{1}{3}}\,dy = \int \frac{9x+6}{x}\,dx$ | |
| | $\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6\ln x \ (+C)$ $\pm k y^{\frac{2}{3}} =$ their (a) | M1 |
| | $\frac{3}{2}y^{\frac{2}{3}} = 9x + 6\ln x \ (+C)$ ft their (a) | A1ft |
| | $y = 8,\ x = 1$ | |
| | $\frac{3}{2}8^{\frac{2}{3}} = 9 + 6\ln 1 + C$ | M1 |
| | $C = -3$ | A1 |
| | $y^{\frac{2}{3}} = \frac{2}{3}(9x + 6\ln x - 3)$ | |
| | $y^2 = (6x + 4\ln x - 2)^3 \quad \left(= 8(3x + 2\ln x - 1)^3\right)$ | A1 (6) |
| | | **[8]** |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| **6.** | (a) $\mathrm{f}(\theta) = 4\cos^2\theta - 3\sin^2\theta$ | |
| | $= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$ | M1 M1 |
| | $= \frac{1}{2} + \frac{7}{2}\cos 2\theta$ * | A1 **(3)** |
| | cso | |
| | (b) $\int \theta \cos 2\theta \, \mathrm{d}\theta = \frac{1}{2}\theta \sin 2\theta - \frac{1}{2}\int \sin 2\theta \, \mathrm{d}\theta$ | M1 A1 |
| | $= \frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\cos 2\theta$ | A1 |
| | $\int \theta \mathrm{f}(\theta) \, \mathrm{d}\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta \sin 2\theta + \frac{7}{8}\cos 2\theta$ | M1 A1 |
| | $\left[ \ \ldots \ \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8}\right] - \left[0 + 0 + \frac{7}{8}\right]$ | M1 |
| | $= \frac{\pi^2}{16} - \frac{7}{4}$ | A1 **(7)** |
| | | **[10]** |

| | | | | |
|---|---|---|---|---|
| **7.** | (a) | $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}8\\13\\-2\end{pmatrix}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$ | or $\overrightarrow{BA}=\begin{pmatrix}-2\\-1\\2\end{pmatrix}$ | M1 |
| | | $\mathbf{r}=\begin{pmatrix}8\\13\\-2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}10\\14\\-4\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ | accept equivalents | M1 A1ft (3) |
| | (b) | $\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}=\begin{pmatrix}10\\14\\-4\end{pmatrix}-\begin{pmatrix}9\\9\\6\end{pmatrix}=\begin{pmatrix}1\\5\\-10\end{pmatrix}$ | or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\10\end{pmatrix}$ | |
| | | $CB=\sqrt{\left(1^2+5^2+(-10)^2\right)}=\sqrt{(126)}\quad(=3\sqrt{14}\approx 11.2)$ | awrt 11.2 | M1 A1 (2) |
| | (c) | $\overrightarrow{CB}.\overrightarrow{AB}=\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{AB}\rvert\cos\theta$ | | |
| | | $(\pm)(2+5+20)=\sqrt{126}\sqrt{9}\cos\theta$ | | M1 A1 |
| | | $\cos\theta=\frac{3}{\sqrt{14}}\Rightarrow\theta\approx 36.7°$ | awrt 36.7° | A1 (3) |
| | (d) |  $\frac{d}{\sqrt{126}}=\sin\theta$ | | M1 A1ft |
| | | $d=3\sqrt{5}(\approx 6.7)$ | awrt 6.7 | A1 (3) |
| | (e) | $BX^2=BC^2-d^2=126-45=81$ | | M1 |
| | | $\triangle CBX=\frac{1}{2}\times BX\times d=\frac{1}{2}\times 9\times 3\sqrt{5}=\frac{27\sqrt{5}}{2}\ (\approx 30.2)$ | awrt 30.1 or 30.2 | M1 A1 (3) |
| | | | | **(14 marks)** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** (a) | $\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$ , | Either of these statements | M1 |
| | $\left(V = 4000h \;\Rightarrow\right) \frac{dV}{dh} = 4000$ | $\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$ | M1 |
| | $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$ | | |
| | Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$<br>or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ | Convincing proof of $\frac{dh}{dt}$ | A1 **AG** |
| | | | **[3]** |
| (b) | When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$ | | |
| | $400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$ | | |
| | From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required | Proof that $k = 0.02$ | B1 **AG** |
| | | | **[1]** |
| (c) | $\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$ | ***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary. | M1 oe |
| | $\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$ | | |
| | time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$ | Correct proof | A1 **AG** |
| | | | **[2]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** (d) | $\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh$ with substitution $h=(20-x)^2$ | | |
| | $\frac{dh}{dx}=2(20-x)(-1)$ or $\frac{dh}{dx}=-2(20-x)$ | Correct $\frac{dh}{dx}$ | B1 aef |
| | $h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$ | | |
| | $\int \frac{50}{20-\sqrt{h}}\,dh = \int \frac{50}{x}.-2(20-x)\,dx$ | $\pm\lambda\int \frac{20-x}{x}\,dx$ or $\pm\lambda\int \frac{20-x}{20-(20-x)}\,dx$ where $\lambda$ is a constant | M1 |
| | $=100\int \frac{x-20}{x}\,dx$ | | |
| | $=100\int \left(1-\frac{20}{x}\right)dx$ | | |
| | $=100(x-20\ln x)\ \ (+c)$ | $\pm\alpha x\pm\beta\ln x\,;\ \alpha,\beta\neq 0$<br>$100x-2000\ln x$ | M1<br>A1 |
| | change limits: when $h=0$ then $x=20$<br>and when $h=100$ then $x=10$<br>$\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh = [100x-2000\ln x]_{20}^{10}$ | | |
| | or $\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$ | Correct use of limits, ie. putting them in the correct way round | |
| | $=(1000-2000\ln 10)-(2000-2000\ln 20)$ | Either $x=10$ and $x=20$ or $h=100$ and $h=0$ | ddM1 |
| | $=2000\ln 20-2000\ln 10-1000$ | Combining logs to give... | |
| | $=2000\ln 2-1000$ | $2000\ln 2-1000$<br>or $-2000\ln\left(\frac{1}{2}\right)-1000$ | A1 **aef** |
| | | | **[6]** |
| (e) | Time required $=2000\ln 2-1000 = 386.2943611...$ sec | | |
| | = 386 seconds (nearest second) | | |
| | = 6 minutes and 26 seconds (nearest second) | <u>6 minutes, 26 seconds</u> | B1 |
| | | | **[1]** |
| | | | **13 marks** |

## Question 1

This question was generally well answered with about 73% of candidates gaining at least 5 of the 7 marks available and about 44% of candidates gaining all 7 marks. Almost all candidates attempted this question with about 13% of them unable to gain any marks.

A significant minority of candidates performed integration by parts the wrong way round in part (a) to give $\frac{1}{3}x^3e^x - \int \frac{1}{3}x^3e^x dx$ and proceeded by attempting to integrate $\frac{1}{3}x^3e^x$. Some candidates failed to realise that integration by parts was required and wrote down answers such as $\frac{1}{3}x^3e^x + c$. Few candidates integrated $e^x$ to give $e^{\frac{1}{2}x^2}$ or applied the product rule of differentiation to give $x^2e^x + 2xe^x$. The majority of candidates, however, were able to apply the first stage of integration by parts to give $x^2e^x - \int 2xe^x dx$. Many candidates realised that they needed to apply integration by parts for a second time in order to find $\int 2xe^x dx$, or in some cases $\int xe^x dx$. Those that failed to realise that a second application of integrating by parts was required either integrated to give the final answer as a two term expression or just removed the integral sign. A significant number of candidates did not organise their solution effectively, and made a bracketing error which often led to a sign error leading to the final incorrect answer of $x^2e^x - 2xe^x - 2e^x + c$.

In part (b), candidates with an incorrect sign in the final term of their integrated expression often proceeded to use the limits correctly to obtain an incorrect answer of $-3e + 2$. Errors in part (b) included not substituting the limit of 0 correctly into their integrated expression; incorrectly dealing with double negatives; evaluating $2e^0$ as 1 or failing to evaluate $e^0$. Most candidates who scored full marks in part (a), achieved the correct answer of $e - 2$ in part (b).

## Question 2

Many candidates had difficulties with the differentiation of the function $u = 2^x$, despite the same problem being posed in the January 2007 paper, with incorrect derivatives of $\frac{du}{dx} = 2^x$ and $\frac{du}{dx} = x2^{x-1}$ being common. Those candidates who differentiated $u$ with respect to $x$ to obtain either $2^x \ln 2$ or $2^x$ often failed to replace $2^x$ with $u$; or if they did this, they failed to cancel the variable $u$ from the numerator and the denominator of their algebraic fraction. Therefore, at this point candidates proceeded to do some "very complicated" integration, always with no chance of a correct solution.

Those candidates who attempted to integrate $k(u+1)^{-2}$ usually did this correctly, but there were a significant number of candidates who either integrated this incorrectly to give $k(u+1)^{-3}$ or $\ln f(u)$.

There were a significant proportion of candidates who proceeded to integrate $u(u+1)^{-2}$ with respect to $x$ and did so by either treating the leading $u$ as a constant or using integration by parts.

Many candidates correctly changed the limits from 0 and 1 to 1 and 2 to obtain their final answer. Some candidates instead substituted $u$ for $2^x$ and used limits of 0 and 1.

## Question 3

At the outset, a significant minority of candidates struggled to extract some or all of the information from the question. These candidates were unable to write down the rate at which this cross-sectional area was increasing, $\frac{dA}{dt} = 0.032$; or the cross-sectional area of the cylinder $A = \pi x^2$ and its derivative $\frac{dA}{dx} = 2\pi x$; or the volume of the cylinder $V = 5\pi x^3$ and its derivative $\frac{dV}{dx} = 15\pi x^2$.

In part (a), some candidates wrote down the volume $V$ of the cylinder as their cross-sectional area $A$. Another popular error at this stage was for candidates to find the curved surface area or the total surface area of a cylinder and write down either $A = 10\pi x^2$ or $A = 12\pi x^2$ respectively. At this stage many of these candidates were able to set up a correct equation to find $\frac{dx}{dt}$ and usually divided 0.032 by their $\frac{dA}{dx}$ and substituted $x = 2$ into their expression to gain 2 out of the 4 marks available. Another error frequently seen in part (a) was for candidates to incorrectly calculate $\frac{0.032}{4\pi}$ as 0.0251. Finally, rounding the answer to 3 significant figures proved to be a problem for a surprising number of candidates, with a value of 0.003 being seen quite often; resulting in loss of the final accuracy mark in part (a) and this sometimes as a consequence led to an inaccurate final answer in part (b).

Part (b) was tackled more successfully by candidates than part (a) – maybe because the chain rule equation $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ is rather more straight-forward to use than the one in part (a). Some candidates struggled by introducing an extra variable $r$ in addition to $x$ and obtained a volume expression such as $V = \pi r^2(5x)$. Many of these candidates did not realise that $r \equiv x$ and were then unable to correctly differentiate their expression for $V$. Other candidates incorrectly wrote down the volume as

$V = 2\pi x^2(5x)$. Another common error was for candidates to state a correct *V*, correctly find $\frac{\mathrm{d}V}{\mathrm{d}x}$, then substitute $x = 2$ to arrive at a final answer of approximately 188.5.

About 10% of candidates were able to produce a fully correct solution to this question.

## Question 4

It was clear to examiners that a significant proportion of candidates found part (i) unfamiliar and thereby struggled to answer this part. Weaker candidates confused the integral of $\ln x$ with the differential of $\ln x$. It was therefore common for these candidates to write down the integral of $\ln x$ as $\frac{1}{x}$, or the integral of $\ln\left(\frac{x}{2}\right)$ as either $\frac{2}{x}$ or $\frac{4}{x}$. A significant proportion of those candidates, who proceeded with the expected by parts strategy, differentiated $\ln\left(\frac{x}{2}\right)$ incorrectly to give either $\frac{2}{x}$ or $\frac{1}{2x}$ and usually lost half the marks available in this part. Some candidates decided from the outset to rewrite $\ln\left(\frac{x}{2}\right)$ as $'\ln x - \ln 2'$, and proceeded to integrate each term and were usually more successful with integrating $\ln x$ than $\ln 2$. It is pleasing to report that a few determined candidates were able to produce correct solutions by using a method of integration by substitution. They proceeded by either using the substitution as $u = \frac{x}{2}$ or $u = \ln\left(\frac{x}{2}\right)$.

A significant minority of candidates omitted the constant of integration in their answer to part (i) and were penalised by losing the final accuracy mark in this part.

In part (ii), the majority of candidates realised that they needed to consider the identity $\cos 2x \equiv 1 - 2\sin^2 x$ and so gained the first method mark. Some candidates misquoted this formula or incorrectly rearranged it. A majority of candidates were then able to integrate $\frac{1}{2}(1 - \cos 2x)$, substitute the limits correctly and arrive at the correct exact answer.

There were, however, a few candidates who used the method of integration by parts in this part, but these candidates were usually not successful in their attempts.

## Question 5

Part (a) of this question proved awkward for many. The integral can be carried out simply by decomposition, using techniques available in module C1. It was not unusual to see integration by parts attempted. This method will work if it is known how to integrate $\ln x$, but this requires a further integration by parts and complicates the question unnecessarily. In part (b), most could separate the variables correctly but the integration of $\dfrac{1}{y^{\frac{1}{3}}}$, again a C1 topic, was frequently incorrect.

Weakness in algebra sometimes caused those who could otherwise complete the question to lose the last mark as they could not proceed from $y^{\frac{2}{3}} = 6x + 4\ln x - 2$ to $y^2 = (6x + 4\ln x - 2)^3$. Incorrect answers, such as $y^2 = 216x^3 + 64\ln x^3 - 8$, were common in otherwise correct solutions.

## Question 6

Candidates tended either to get part (a) fully correct or make no progress at all. Of those who were successful, most replaced the $\cos^2\theta$ and $\sin^2\theta$ directly with the appropriate double angle formula. However many good answers were seen which worked successfully via $7\cos^2\theta - 3$ or $4 - 7\sin^2\theta$.

Part (b) proved demanding and there were candidates who did not understand the notation $\theta\,\mathrm{f}(\theta)$. Some just integrated $\mathrm{f}(\theta)$ and others thought that $\theta\,\mathrm{f}(\theta)$ meant that the argument $2\theta$ in $\cos 2\theta$ should be replaced by $\theta$ and integrated $\frac{1}{2}\theta + \frac{7}{2}\theta\cos\theta$. A few candidates started by writing $\int \theta\,\mathrm{f}(\theta)\,\mathrm{d}\theta = \theta\int \mathrm{f}(\theta)\,\mathrm{d}\theta$, treating $\theta$ as a constant. Another error seen several times was $\int \theta\,\mathrm{f}(\theta)\,\mathrm{d}\theta = \int\left(\frac{1}{2}\theta + \frac{7}{2}\cos 2\theta^2\right)\mathrm{d}\theta$.

Many candidates correctly identified that integration by parts was necessary and most of these were able to demonstrate a complete method of solving the problem. However there were many errors of detail, the correct manipulation of the negative signs that occur in both integrating by parts and in integrating trigonometric functions proving particularly difficult. Only about 15% of candidates completed the question correctly.

## Question 7

This proved the most demanding question on the paper. Nearly all candidates could make some progress with the first three parts but, although there were many, often lengthy attempts, success with part (d) and (e) was uncommon. Part (a) was quite well answered, most finding *AB* or *BA* and writing down *OA*+$\lambda$*AB*, or an equivalent. An equation does, however need an equals sign and a subject and many lost the final A mark in this part by omitting the "$\mathbf{r} =$" from, say, $\mathbf{r} = 8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. In part (b), those who realised that a magnitude or length was required were usually successful. In part (c), nearly all candidates knew how to evaluate a scalar product and obtain an equation in $\cos\theta$, and so gain the method marks, but the vectors chosen were not always the right ones and a few candidates gave the obtuse angle. Few made any real progress with parts (d) and (e). As has been stated in previous reports, a clear diagram helps a candidate to appraise the situation and choose a suitable method. In this case, given the earlier parts of the question, vector methods, although possible, are not really appropriate to these parts, which are best solved using elementary trigonometry and Pythagoras' theorem. Those who did attempt vector methods were often very unclear which vectors were perpendicular to each other and, even the minority who were successful, often wasted valuable time which sometimes led to poor attempts at question 8. It was particularly surprising to see quite a large number of solutions attempting to find a vector, *CX* say, perpendicular to *l*, which never used the coordinates or the position vector of *C*.

## Question 8

This proved by far the most difficult question on the paper and discriminated well for those candidates who were above the grade *A* threshold for this paper. Only a few candidates were able to score above 8 or 9 marks on this question.

Many 'fudged' answers were seen in part (a). A more rigorous approach using the chain rule of $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ was required, with candidates being expected to state $\frac{dV}{dt}$ and $\frac{dV}{dh}$ (or its reciprocal). The constant of proportionality also proved to be a difficulty in this and the following part.

Few convincing proofs were seen in part (b) with a significant number of candidates not understanding how to represent $400 \text{ cm}^3 \text{ s}^{-1}$ algebraically.

Only a minority of candidates were able to correctly separate the variables in part (c). Far too often, expressions including $\int \frac{dh}{0.4} = \int 0.02\sqrt{h}\, dt$ were seen by examiners. There were a significant number of candidates who having written $\int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$ could not progress to the given answer by multiplying the integral on the left hand side by $\frac{50}{50}$.

Despite struggling with the previous three parts, a majority of candidates were able to attempt part (d), although only a few candidates were able to produce the correct final exact answer. A majority of candidates who attempted this part managed to correctly obtain $\frac{dh}{dx} = 2x - 40$ and then use this and the given substitution to write down an integral in *x*. At this point a significant number of candidates were unable to manipulate the expression $k\left(\frac{x-10}{x}\right)$ into an expression of the form $k\left(1 - \frac{20}{x}\right)$. The converted limits $x = 10$ and $x = 20$, caused an added problem for those candidates who progressed further, with a significant number of candidates incorrectly applying $x = 10$ as their lower limit and $x = 20$ as their upper limit.

A time of 6 minutes 26 seconds was rarity in part (e).

## Statistics for C4 Practice Paper G4

| | | | | Mean score for students achieving grade: | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Qu** | **Max score** | **Modal score** | **Mean %** | **ALL** | **A*** | **A** | **B** | **C** | **D** | **E** | **U** |
| **1** | 7 | 7 | 69 | 4.83 | 6.81 | 6.31 | 5.42 | 4.11 | 2.74 | 1.62 | 0.74 |
| **2** | 6 | | 35 | 2.09 | | 3.51 | 1.66 | 0.88 | 0.46 | 0.20 | 0.09 |
| **3** | 8 | | 36 | 2.89 | | 4.90 | 2.22 | 1.12 | 0.56 | 0.27 | 0.12 |
| **4** | 9 | | 50 | 4.50 | | 6.53 | 4.29 | 2.57 | 2.15 | 0.50 | 0.18 |
| **5** | 8 | | 54 | 4.28 | | 6.10 | 3.27 | 2.02 | 1.15 | 0.45 | 0.29 |
| **6** | 10 | | 44 | 4.38 | 9.04 | 6.25 | 3.83 | 2.06 | 1.03 | 0.44 | 0.17 |
| **7** | 14 | | 41 | 5.75 | | 8.24 | 5.36 | 3.73 | 2.46 | 1.55 | 0.89 |
| **8** | 13 | | 27 | 3.50 | | 5.79 | 2.58 | 1.05 | 0.69 | 0.31 | 0.15 |
| | **75** | | **43** | **32.22** | | **47.63** | **28.63** | **17.54** | **11.24** | **5.34** | **2.63** |