

Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Gold Level G1

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
61	53	43	35	27	21

1. A lorry of mass 2000 kg is moving down a straight road inclined at angle α to the horizontal, where $\sin \alpha = \frac{1}{25}$. The resistance to motion is modelled as a constant force of magnitude 1600 N. The lorry is moving at a constant speed of 14 m s^{-1} .

Find, in kW, the rate at which the lorry's engine is working.

(6)

2. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P . The coefficient of restitution between P and Q is e .

(a) Find the speed of Q immediately after the collision.

(6)

Given that the direction of motion of P is reversed by the collision,

(b) find the range of possible values of e .

(5)

3. A particle P moves on the x -axis. At time t seconds the velocity of P is $v \text{ m s}^{-1}$ in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \quad t \geq 0$$

Find

(a) the times when P is instantaneously at rest,

(3)

(b) the greatest speed of P in the interval $0 \leq t \leq 4$,

(5)

(c) the total distance travelled by P in the interval $0 \leq t \leq 4$.

(5)

4.

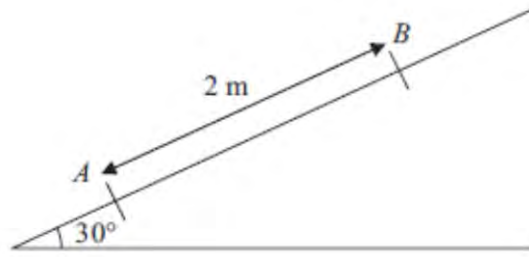


Figure 2

A particle P of mass 0.5 kg is projected from a point A up a line of greatest slope AB of a fixed plane. The plane is inclined at 30° to the horizontal and $AB = 2$ m with B above A , as shown in Figure 2. The particle P passes through B with speed 5 m s^{-1} . The plane is smooth from A to B .

(a) Find the speed of projection.

(4)

The particle P comes to instantaneous rest at the point C on the plane, where C is above B and $BC = 1.5$ m. From B to C the plane is rough and the coefficient of friction between P and the plane is μ .

By using the work-energy principle,

(b) find the value of μ .

(6)

5.

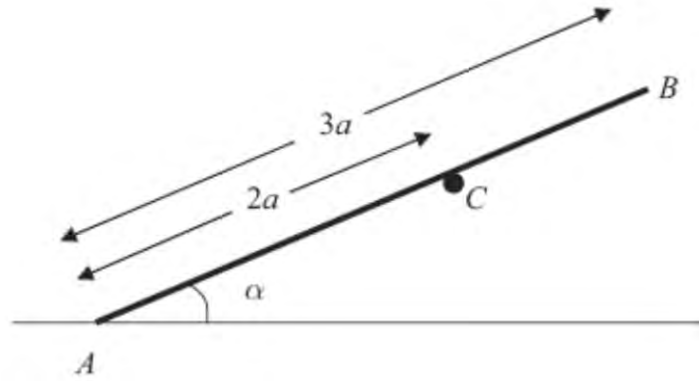


Figure 2

A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB . The rod has length $3a$ and weight W and rests on the peg at C , where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in Figure 2.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3 \cos^2 \alpha)W$. (6)

Given that the rod is in limiting equilibrium and that $\cos \alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground. (5)

6.

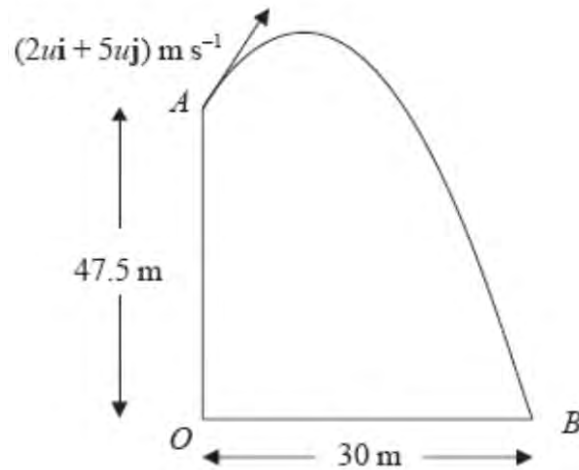


Figure 3

[In this question, the unit vectors \mathbf{i} and \mathbf{j} are in a vertical plane, \mathbf{i} being horizontal and \mathbf{j} being vertical.]

A particle P is projected from the point A which has position vector $47.5\mathbf{j}$ metres with respect to a fixed origin O . The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j}) \text{ m s}^{-1}$. The particle moves freely under gravity passing through the point B with position vector $30\mathbf{i}$ metres, as shown in Figure 3.

- (a) Show that the time taken for P to move from A to B is 5 s . (6)
- (b) Find the value of u . (2)
- (c) Find the speed of P at B . (5)
-

7.

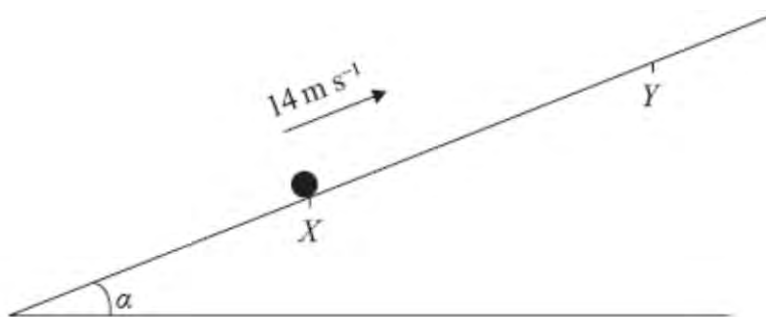


Figure 4

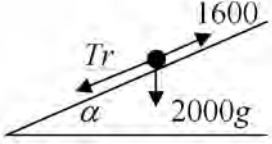
A particle P of mass 2 kg is projected up a rough plane with initial speed 14 m s^{-1} , from a point X on the plane, as shown in Figure 4. The particle moves up the plane along the line of greatest slope through X and comes to instantaneous rest at the point Y . The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{7}{24}$. The coefficient of friction between the particle and the plane is $\frac{1}{8}$.

(a) Use the work-energy principle to show that $XY = 25 \text{ m}$. (7)

After reaching Y , the particle P slides back down the plane.

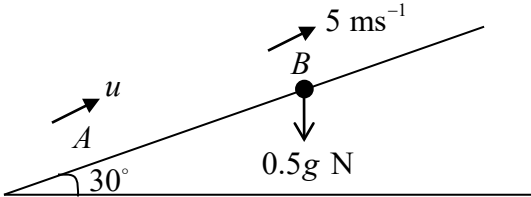
(b) Find the speed of P as it passes through X . (4)

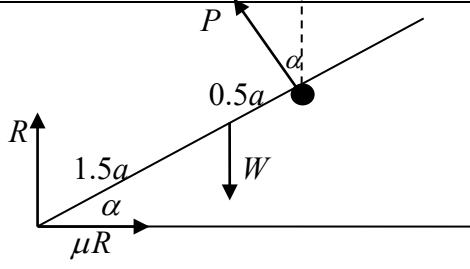
TOTAL FOR PAPER: 75 MARKS

Question Number	Scheme	Marks
1.	 <p>Resolve \nearrow: $T_r + 2000g \times \sin \alpha = 1600$ $(T_r = 816)$ $P = 816 \times 14 \text{ (W)}$ ft their T_r $\approx 11 \text{ (kW)}$ accept 11.4</p>	<p>M1 A1 A1 M1 A1ft A1 cso</p> <p>[6]</p>

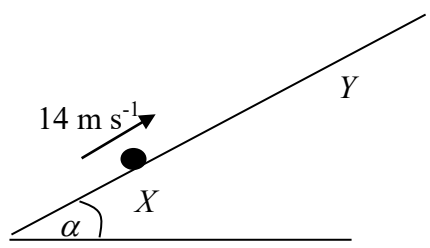
2.	<p>(a) $3m \cdot 2u - 4mu = 3mv_1 + 4mv_2$ $e(2u + u) = -v_1 + v_2$ $\frac{u(2+9e)}{7} = v_2$</p> <p>(b) $v_1 = \frac{2u(1-6e)}{7}$ $v_1 < 0 \Rightarrow e > \frac{1}{6}$ $1 \geq e > \frac{1}{6}$</p>	<p>M1 A1 M1 A1 DM1 A1 (6) M1 A1 DM1 A1 B1 (5) [11]</p>
----	---	---

3	a	$v = 0 = 2t^2 - 14t + 20$ $= 2(t-2)(t-5)$ $t = 2 \text{ or } t = 5$	M1 M1 A1	(3)
	b	$(t = 0), v = 20 \text{ (m s}^{-1}\text{)}$ $a = 4t - 14 = 0$ $t = \frac{7}{2}, v = 2 \times \frac{3}{2} \times \frac{-3}{2} = \frac{-9}{2}$ Max speed = 20 ms^{-1}	B1 M1 M1A1 A1	(5)
	c	$\int 2t^2 - 14t + 20 dt = \frac{2}{3}t^3 - 7t^2 + 20t (+C)$ $\text{Distance} = \left[\frac{2}{3}t^3 - 7t^2 + 20t \right]_0^2 - \left[\frac{2}{3}t^3 - 7t^2 + 20t \right]_2^4$ $= 2 \times \left[\frac{2}{3}t^3 - 7t^2 + 20t \right]_2^4$ $= 2 \times \left[\frac{16}{3} - 7 \times 4 + 40 \right] - \left[\frac{2 \times 64}{3} - 7 \times 16 + 80 \right] = 24 \text{ (m)}$	M1 A1 M1 A1 A1	(5) [13]

Question Number	Scheme	Marks
4. (a)	 <p data-bbox="284 607 778 678"> $0.5g \times 2 \sin 30 = \frac{1}{2} \times 0.5u^2 - \frac{1}{2} \times 0.5 \times 5^2$ </p> <p data-bbox="284 689 603 761"> $\frac{1}{4}u^2 = 0.5g + \frac{1}{2} \times 0.5 \times 5^2$ </p> <p data-bbox="284 779 646 824"> $u = 6.7 \text{ m s}^{-1}$ (accept 6.68) </p>	<p data-bbox="1294 622 1390 656">M1 A1</p> <p data-bbox="1294 786 1414 819">DM1 A1</p> <p data-bbox="1433 842 1469 875">(4)</p>
(b)	<p data-bbox="284 927 485 963">$R = 0.5g \cos 30$</p> <p data-bbox="284 974 533 1010">$F = 0.5g \cos 30 \times \mu$</p> <p data-bbox="284 1021 667 1057">Work done by friction = $1.5F$</p> <p data-bbox="284 1077 762 1149"> $\frac{1}{2} \times 0.5 \times 5^2 = 1.5F + 0.5g \times 1.5 \sin 30$ </p> <p data-bbox="284 1169 707 1240"> $\mu = \frac{\frac{1}{2} \times 0.5 \times 5^2 - 0.5g \times 1.5 \sin 30}{0.5g \cos 30 \times 1.5}$ </p> <p data-bbox="284 1261 715 1296"> $\mu = 0.40$ (accept 0.4 or 0.405) </p>	<p data-bbox="1294 931 1334 965">B1</p> <p data-bbox="1294 976 1334 1010">M1</p> <p data-bbox="1294 1099 1437 1133">M1 A1 A1</p> <p data-bbox="1294 1267 1334 1301">A1</p> <p data-bbox="1433 1312 1469 1346">(6)</p> <p data-bbox="1414 1357 1469 1391">[10]</p>

Question Number	Scheme	Marks
5. (a)	 <p style="text-align: center;">$R(\uparrow) \quad R + P \cos \alpha = W$</p> <p style="text-align: center;">$M(A) \quad P \times 2a = W \times 1.5a \cos \alpha$</p> <p style="text-align: center;">$\left(P = \frac{3}{4} W \cos \alpha \right)$</p> <p style="text-align: center;">$R = W - P \cos \alpha = W - \frac{3}{4} W \cos^2 \alpha$</p> <p style="text-align: center;">$= \frac{1}{4} (4 - 3 \cos^2 \alpha) W \quad *$</p>	<p style="text-align: right;">M1 A1</p> <p style="text-align: right;">M1 A1</p> <p style="text-align: right;">DM1</p> <p style="text-align: right;">cso A1</p> <p style="text-align: right;">(6)</p>
(b)	<p>Using $\cos \alpha = \frac{2}{3}$, $R = \frac{2}{3} W$</p> <p>$R(\rightarrow) \quad \mu R = P \sin \alpha$</p> <p>Leading to $\mu = \frac{3}{4} \sin \alpha$</p> <p>$\left(\sin \alpha = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3} \right)$</p> <p style="text-align: center;">$\mu = \frac{\sqrt{5}}{4}$</p>	<p style="text-align: right;">B1</p> <p style="text-align: right;">M1 A1</p> <p style="text-align: right;">awrt 0.56 M1 A1</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[11]</p>

Question Number	Scheme	Marks
6. (a)	$\rightarrow 30 = 2ut$ $\uparrow -47.5 = 5ut - 4.9t^2$ $-47.5 = 75 - 4.9t^2 \quad \text{eliminating } u \text{ or } t$ $t^2 = \frac{75 + 47.5}{4.9} (= 25)$ $t = 5 \quad *$ <p style="text-align: right;">cso</p>	B1 M1 A1 DM1 DM1 A1 (6)
(b)	$30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$	M1 A1 (2)
(c)	$\uparrow \quad \dot{y} = 5u - 9.8t = -34 \quad \text{M1 requires both}$ $\rightarrow \quad \dot{x} = 2u = 6 \quad \dot{x} \text{ and } \dot{y}$ $v^2 = 6^2 + (-34)^2$ $v \approx 34.5 \text{ (ms}^{-1}\text{)} \quad \text{accept 35}$	M1 A1 A1 DM1 A1 (5)
Alternative to (c)		[13]
$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m \times g \times 47.5 \quad \text{with} \quad v_A^2 = 6^2 + 15^2 = 261$ $v_B^2 = 261 + 2 \times 9.8 \times 47.5 (= 1192)$ $v_B \approx 34.5 \text{ (ms}^{-1}\text{)} \quad \text{accept 35}$		M1 A(2,1,0) DM1 A1 (5)
BEWARE : Watch out for incorrect use of $v^2 = u^2 + 2as$		

<p>7.</p> <p>(a)</p>	 <p>KE at X = $\frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 14^2$</p> <p>GPE at Y = $mgd \sin \alpha \left(= 2 \times g \times d \times \frac{7}{25} \right)$</p> <p>Normal reaction $R = mg \cos \alpha$</p> <p>Friction = $\mu \times R = \frac{1}{8} \times 2g \times \frac{24}{25}$</p> <p>Work Energy: $\frac{1}{2}mv^2 - mgd \sin \alpha = \mu \times R \times d$ or equivalent</p> $196 = \frac{14gd}{25} + \frac{6gd}{25} = \frac{20gd}{25}$ $d = 25 \text{ m}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1A1</p> <p>A1</p> <p>(7)</p>
<p>(b)</p>	<p>Work Energy</p> <p>First time at X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2$</p> <p>Work done = $\mu \times R \times 2d = \frac{1}{8} \times 2g \times \frac{24}{25} \times 2d$</p> <p>Return to X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8} \times 2g \times \frac{24}{25} \times 50$</p> $v = 8.9 \text{ ms}^{-1} \quad (\text{accept } 8.85 \text{ ms}^{-1})$ <p>OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$</p> $2a = 2g \sin \alpha - F_{\max} = 2g \times \frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$ $v^2 = (0+)^2 + 2 \times a \times s = 8g ; v = 8.9 \quad (\text{accept } 8.85 \text{ ms}^{-1})$	<p>M1A1</p> <p>DM1A1</p> <p>(4)</p> <p>M1A1</p> <p>DM1;A1</p> <p>[11]</p>

Examiner reports

Question 1

This was a straightforward question for many candidates. However, as usual, some were not sure how to deal with power and velocity in a question involving forces and the inclined plane. A surprising number of candidates felt that the lorry had to work against gravity when it was going down the hill. Too many candidates failed to realise that using 9.8 for gravity could not give answers to a high degree of accuracy. Only answers to 2 or 3 significant figures were accepted.

Question 2

In part (a) many candidates gave correct equations for conservation of momentum and the impact law. If there were errors these were usually sign errors in the impact law, with many having the speed of approach as u rather than $3u$. It was more common to find errors when candidates were using a formulaic approach rather than thinking of "speed of approach" and "speed of separation". Some candidates struggled to use their simultaneous equations to find an expression for the speed of Q , often going a very long way round to get to their answer.

For part (b), having found an expression for the velocity of P , many candidates did not realise that the question simply required them to use sign as an indicator of direction. Some thought that they needed to consider the relative velocities of P and Q , and several went back to the beginning of the question and repeated all their working having changed their initial assumption about the direction of motion of P after the impact. Some candidates with correct solutions lost the final mark because they did not consider the upper limit to the range of possible values for e .

Question 3

Fully correct solutions to this question were rare, although most candidates made a correct start to each part.

Part (a) was usually fully correct.

The majority of candidates did not appreciate the subtlety of part (b), not realising the difference between a maximum and a local maximum. Those whose solution was accompanied by a sketch were most likely to reach a correct solution. Candidates who started by finding the local minimum velocity (maximum speed) frequently went no further. Candidates who started by finding values of velocity for different values of time often discarded the negative values without realising that they needed to be comparing the magnitudes of their values.

In part (c) the difference between displacement and distance was frequently either ignored or went unrecognised. Despite the hint given by part (a), the majority of candidates did not consider the actual motion of the particle and did not recognise that the particle was re-tracing its previous path and that they must split the integral to calculate distance rather than displacement.

Question 4

It was pleasing to see that an energy approach was the favoured method in part (a), with only a minority opting to use *suvat* equations. Of those using energy, several candidates had the kinetic energies the wrong way round in their equation and appeared not to notice that the velocity of projection at the bottom of the plane was less than the velocity after the particle had risen 2 metres up the plane. Some candidates used the distance along the plane instead of vertical height when calculating the gain in potential energy. Most candidates using *suvat*, correctly deduced that the acceleration was $-\frac{g}{2}$ and proceeded to the correct solution.

In part (b) some candidates ignored the instruction to use the work-energy principle for this part, and lost marks as a result. Many correct solutions were seen. The most common error in the energy equation was to double count the gain in potential energy, treating it as part of the increase in energy and again as a term in work done against 'resistive forces'. Some candidates considered the energy change from *A* to *C* but forgot that the friction only acted over the final 1.5 m, not the full 3.5 m. There was evidence of some confusion between force and energy, with some candidates equating the friction, rather than the work done against the friction, to the change in energy.

Question 5

This proved to be the most challenging question on the paper. It turned out to be a question which differentiated well between those who had a thorough understanding of the principles and those with a relatively poor grasp. As usual, too many solutions were spoiled by candidates who were not able to find the correct components of their forces, confusing $\cos \alpha$ and $\sin \alpha$.

(a) Candidates who took moments about *A* and resolved vertically were the most successful with many producing compact and complete solutions. Others taking moments about *C* or *B* invariably omitted a force or were not able to deal with the extra unknowns.

Common errors were in mistakes with the reactions at *A* and *C*, e.g. a vertical reaction at *C*, the reaction at *A* perpendicular to the rod, and use of horizontal and vertical components at *C* with no method for combining them. Several candidates had the weight of the plank acting at *C*. Also, some introduced a reaction at *B*, usually horizontal, perhaps through confusion with problems relating to a ladder against a wall.

(b) This was answered a little more successfully with good use of the given answer from part (a). Most candidates seemed confident in their use of $F = \mu R$ although some confused the reaction at *A* and the reaction at *C*. This confusion was sometimes the consequence of poorly labelled diagrams. There was some difficulty in finding and using the value for $\sin \alpha$.

Question 6

This question proved to be very challenging for many candidates. The best candidates worked through swiftly and efficiently scoring full marks in a relatively short solution. A number eliminated t rather than u in part (a), thus finding u first but this did not cause any great difficulty.

Unfortunately, several candidates were completely unprepared to deal with the initial velocity in vector form. Many of these went on to recombine the components and find a speed and angle of projection, then faithfully worked on with sin/cos and $\sqrt{29}$ without realising that it came back to 2 and 5! The candidates who made the least progress were those who tried to use the equations of motion with the velocity in complete vector form and the displacement and acceleration as scalars.

Although it created considerable extra work, and invariably went wrong, a small proportion of candidates tried to break the task down by working from the point of projection to the highest point and then from the highest point to B . Several candidates who arrived at a correct equation in u and t then went on to apply the quadratic formula inappropriately to obtain an expression for t in terms of u . A number also got the wrong answer for t in part (a) but did not then pick up the given answer of $t=5$; they persevered with their incorrect result and hence lost many more marks.

In part (c) we encountered all the usual errors, although it was good to find far fewer candidates making inappropriate use of $v^2 = u^2 + 2as$. Most students understood the need to find both the horizontal and the vertical component of the velocity at B . Several candidates were not sufficiently clear about the direction of motion, and made errors due to confusion over signs. A minority of candidates used conservation of energy without being required to do so, and were usually successful. Too many candidates lost the final mark due to an inappropriate level of accuracy in their final answers.

Question 7

Part (c) (i) was done successfully by most good candidates, but part (c)(ii) had a very low success rate, with most candidates simply equating the \mathbf{i} and \mathbf{j} components of \mathbf{PQ} . This proved to be a good discriminator at the end of the paper.

In this question a significant minority tried to find the acceleration in part (a), despite being instructed to use work and energy. For those candidates using the required method, the most common error was to include the work done against gravity with the work done against the friction but then also include the change in gravitational potential energy, thus double counting. Although many answered successfully, there were some who rather "fudged" the arrival at the given answer. A common error was to assume that the vertical height gained was 7, and this lead inevitably to the correct answer without the necessary algebraic treatment of the situation. The most common approach to part (b) was to revert to the use of $F = ma$, rather than sticking to the work-energy principle, for which they had already done most of the work in part (a). Many who attempted a work-energy approach forgot to include the work done against friction in this part, even though they had used it in part (a). Very few candidates used energy and point X as the initial and final position to find the answer.

Statistics for M2 Practice Paper Gold 1

Mean average scored by candidates achieving grade:

Qu	Max Score	Modal score	Mean %	ALL	A*	A	B	C	D	E	U
1	6		80.7	4.84		5.39	4.98	4.58	4.11	3.45	2.43
2	11		69.8	7.68	9.52	8.75	7.54	6.56	5.38	4.44	2.84
3	13	8	61.3	7.97	9.72	8.72	7.68	7.01	6.46	5.87	4.84
4	10		69.2	6.92	9.05	8.14	6.51	5.26	3.99	2.84	1.89
5	11		48.8	5.37		8.25	4.40	2.67	1.50	0.93	0.47
6	13		62.3	8.10		10.31	6.32	2.94	1.64	0.74	0.72
7	11		64.4	7.08		8.89	6.50	5.07	4.15	3.07	1.79
	75		64.0	47.96		58.45	43.93	34.09	27.23	21.34	14.98