GCE Examinations

Decision Mathematics
Module D1

Advanced Subsidiary / Advanced Level

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

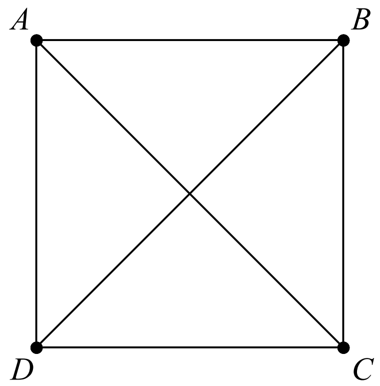
Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Dave Hayes

1. *(a)* Make plane drawings of each of the graphs shown in Figure 1.

Graph 1



Graph 2

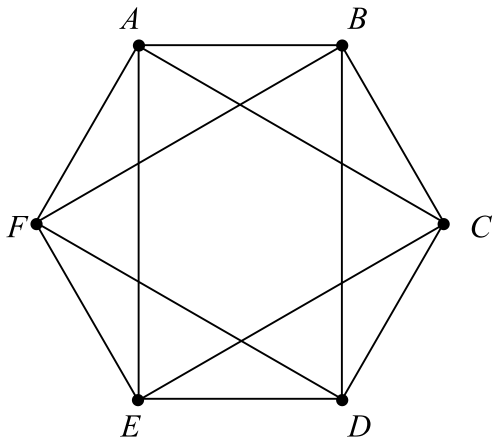


Fig. 1 **(3 marks)**

(b) State the name given to Graph 1 and write down the features that identify it.
(2 marks)

(c) State, with a reason, whether it is possible to add further arcs to Graph 2 such that it remains a simple connected graph. No further vertices may be added.
(1 mark)

2. *This question should be answered on the sheet provided.*

A builder is going to put up houses on a plot of land of area 12 000 m^2.

There are 5 designs to choose from and no more than one of each design can be built.

Design	Kendal	Milverton	Arlington	Elford	Grosvenor
Plot area ('000 m^2)	3	11	3	5	10
Value (£'000 s)	100	190	40	80	120

The builder needs to work out which houses he should build in order to maximise the total value of the site. He does this using a tree diagram and each "branch" on the tree is terminated when the total area of land on that branch exceeds 12 000 m^2.

(a) (i) Complete the tree diagram so that each branch is terminated or all choices have been considered.

(ii) Hence, determine which designs the builder should use and the total value that the site will have when completed.

(6 marks)

(b) Explain how this method could be altered if more than one of each design is allowed.

(1 mark)

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3. *(a)* Draw a graph with 6 vertices, each of degree 1. **(1 mark)**

(b) Draw two graphs with 6 vertices, each of degree 2 such that:

(i) the graph is connected,

(ii) the graph is not connected. **(2 marks)**

A simple connected graph has 5 vertices each of degree x.

(c) Find the possible values of x and explain your answer. **(2 marks)**

(d) For each value of x you found in part *(c)* draw a possible graph. **(2 marks)**

Turn over

4. A company produces x_1 finished articles at the end of January,
x_2 finished articles at the end of February,
x_3 finished articles at the end of March,
x_4 finished articles at the end of April.

Other details for each month are as follows:

Month	January	February	March	April
Demand at end of month	200	350	250	200
Production costs per article	£1000	£1800	£1600	£1900

The cost of storing each finished but unsold article is £500 per month. Thus, for example, any article unsold at the end of January would incur a £500 charge if it is stored until the end of February or a £1000 charge if it is stored until the end of March.

There must be no unsold stock at the end of April.

The selling price of each article is £4000 and the total profit (£P) must be maximised.

(a) Rewrite x_4 in terms of the other 3 variables. **(1 mark)**

(b) Show that the total cost incurred (£C) is given by:

$$C = 600x_1 + 900x_2 + 200x_3 + 1\,125\,000.$$ **(5 marks)**

(c) Hence, show that $P = {}^{-}600x_1 - 900x_2 - 200x_3 + 2\,875\,000$. **(1 mark)**

(d) Three of the constraints operating can be expressed as $x_1 \geq 200$, $x_2 \geq 0$ and $x_3 \geq 0$. Write down inequalities representing two further constraints.
(2 marks)

(e) Explain why it is not appropriate to use a graphical method to solve this problem.

(1 mark)

(f) An employee of the company wishes to use the Simplex algorithm to solve the problem. He tries to generate an initial tableau with x_1, x_2 and x_3 as the non-basic variables.

Explain why this is not appropriate and explain what he should do instead. You are not required to generate an initial tableau or to solve the problem.
(2 marks)

5. *This question should be answered on the sheet provided.*

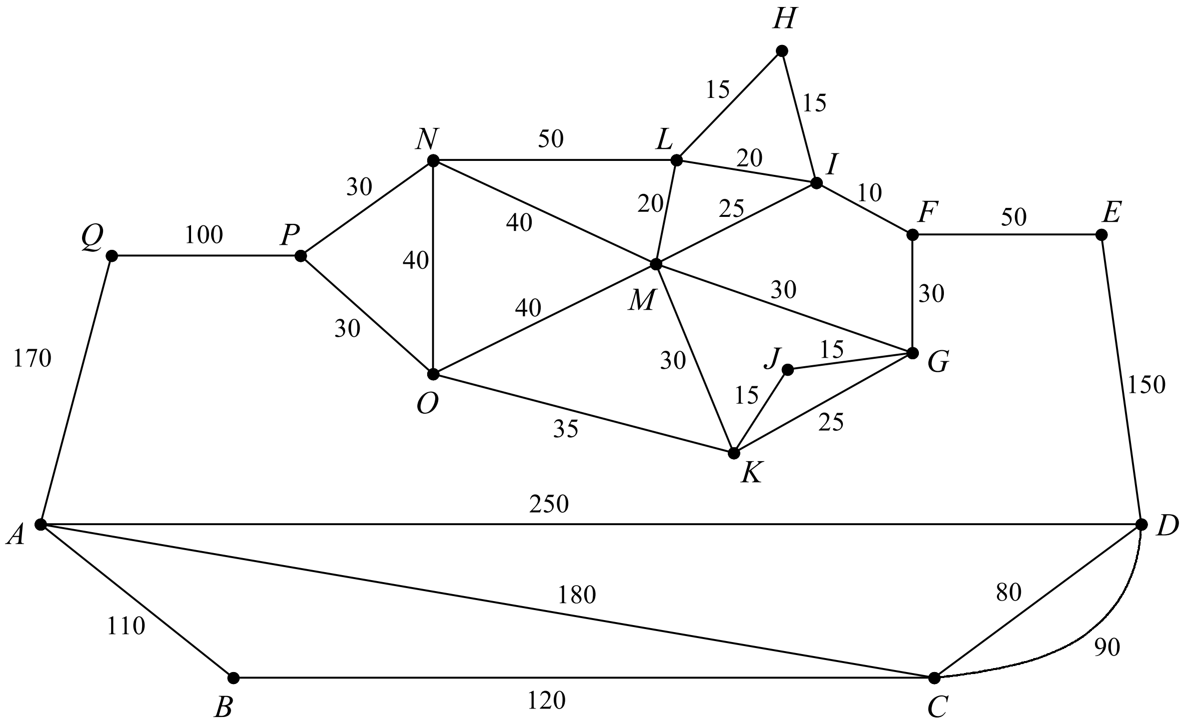


Fig. 2

Figure 2 shows a weighted network representing the paths in a certain part of St. Andrews. The numbers on the arcs represent the lengths of the paths in metres.

(a) Use Dijkstra's algorithm to find a route of minimum length from *P* to *F*. You do not need to consider routes via vertex *Q*.

You must show clearly:

(i) the order in which you labelled the vertices,

(ii) how you found a route of minimum length from your labelling. **(8 marks)**

Each night a security guard walks along each of the paths in Figure 2 at least once.

(b) The security office is at vertex *A*, so she must start and finish her inspection at *A*. Find the minimum distance that she must walk each night.

(4 marks)

Turn over

6. *This question should be answered on the sheet provided.*

A town has adopted a one-way system to cope with recent problems associated with congestion in one area.

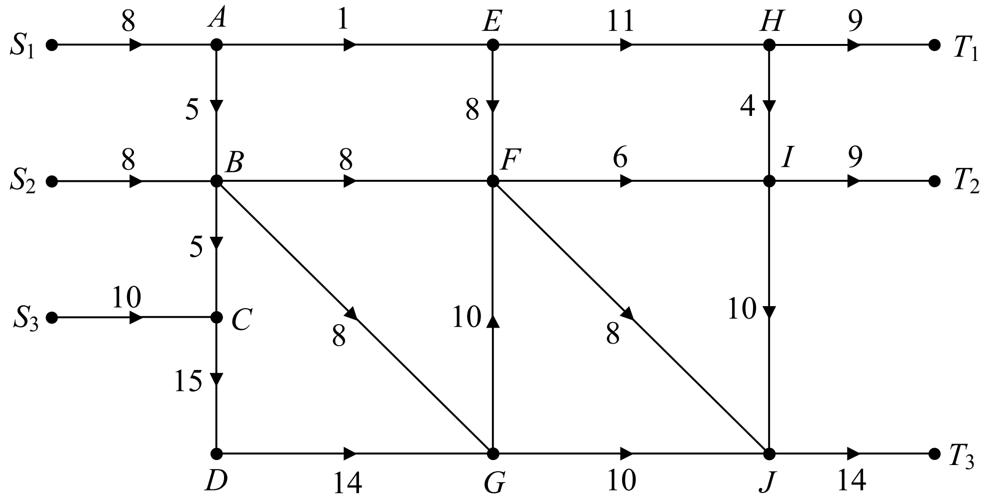


Fig. 3

Figure 3 models the one-way system as a capacitated directed network. The numbers on the arcs are proportional to the number of vehicles that can pass along each road in a given period of time.

(a) Find the capacity of the cut which passes through the arcs *AE*, *BF*, *BG* and *CD*.

(2 marks)

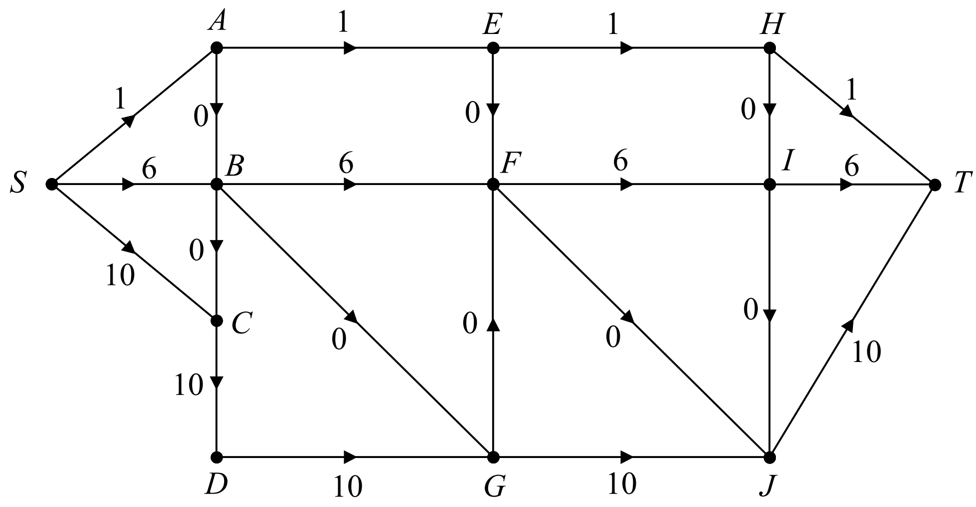


Fig. 4

Figure 4 shows a feasible flow of 17 through the same network. For convenience, a supersource, *S*, and a supersink, *T*, have been used.

(b) (i) Use the labelling procedure to find the maximum flow through this network. You must list each flow-augmenting route you use together with its flow.

(ii) Show your maximum flow pattern and state its value. **(7 marks)**

(c) Prove that your flow is the maximum possible through the network. **(2 marks)**

(d) It is suggested that the maximum flow through the network could be increased by making road *EF* undirected, so that it has a capacity of 8 in either direction.

Using the maximum flow-minimum cut theorem, find the increase in maximum flow this change would allow.

(2 marks)

(e) An alternative suggestion is to widen a single road in order to increase its capacity. Which road, on its own, could lead to the biggest improvement, and what would be the largest maximum flow this could achieve.

(2 marks)

Turn over

7. A project involves six tasks, some of which cannot be started until others have been completed. This is shown in the table below.

Task	Duration (minutes)	Immediate predecessors
A	18	–
B	23	–
C	13	A, B
D	9	A
E	28	B, D
F	23	C

(a) Draw an activity network for this project. **(4 marks)**

(b) By labelling your network, find the critical path and the minimum duration of the project.
(3 marks)

(c) Find the float time of each non-critical activity. **(3 marks)**

An extra condition is now imposed. Task A may not begin until task B has been underway for at least 10 minutes.

(d) Draw a new network taking into account this restriction. **(3 marks)**

(e) Find a revised value for the minimum duration of the project and state the new critical path.
(3 marks)

END

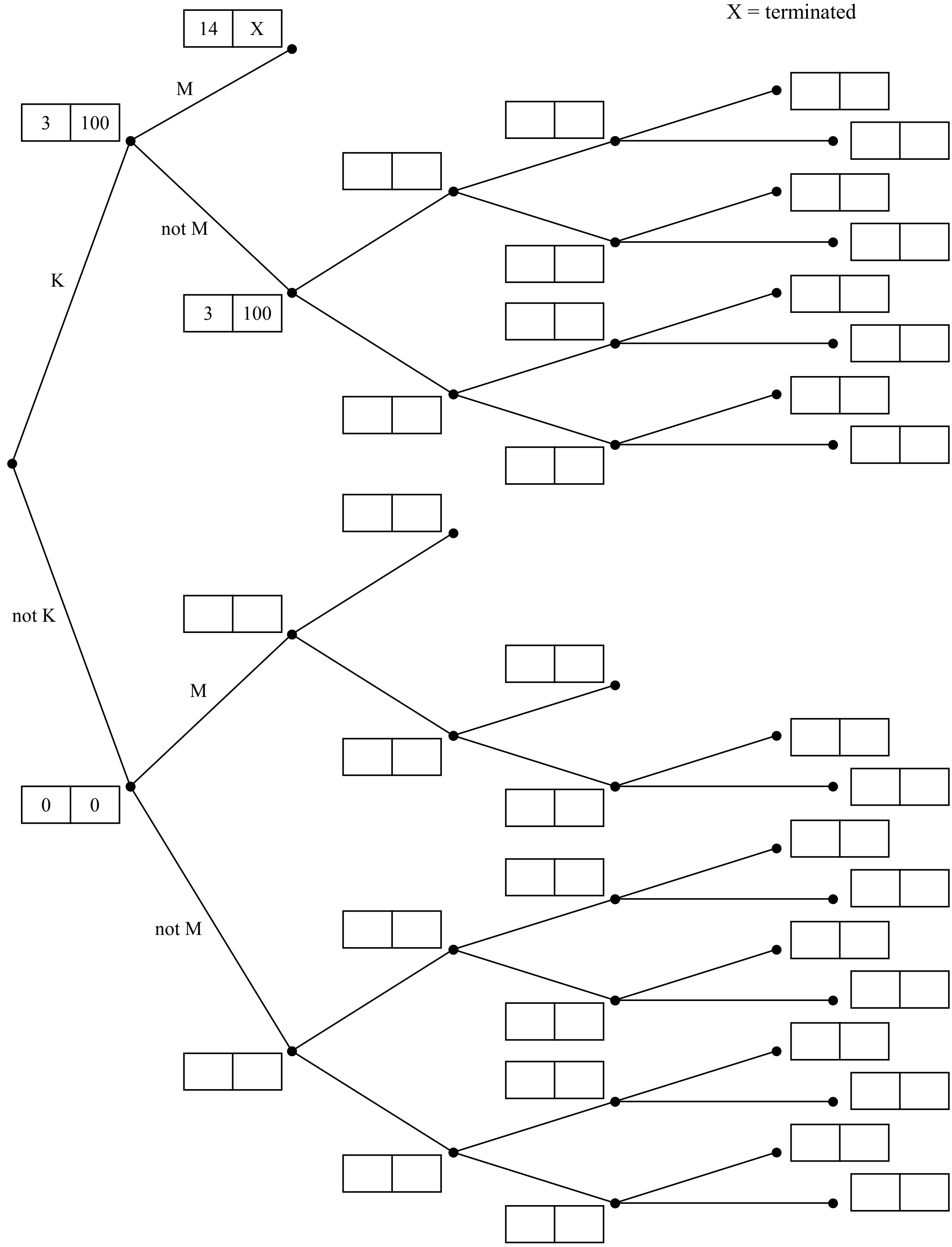
NAME ....

Sheet for answering question 2

Please hand this sheet in for marking

KEY: | Area of land ('000 m²) | Value (£ '000s) |

X = terminated



(a) ....

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(b) ....

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Sheet for answering question 5 *NAME* ……………………………………….

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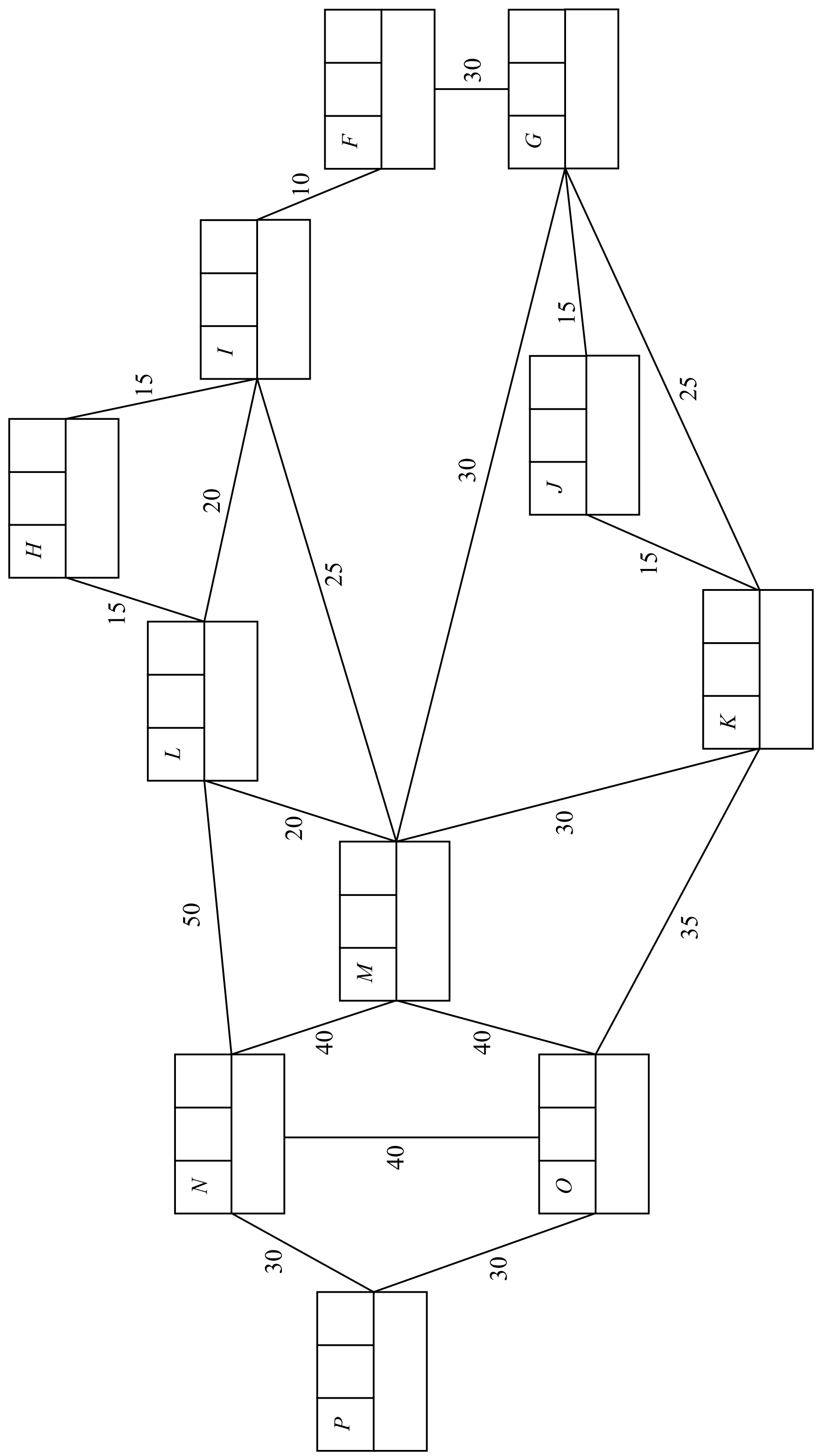
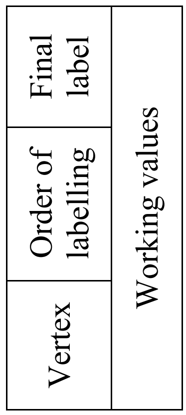
(a)



KEY:



(b) …………………………………………………………………………

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Sheet for answering question 6 *NAME* ……………………………………….

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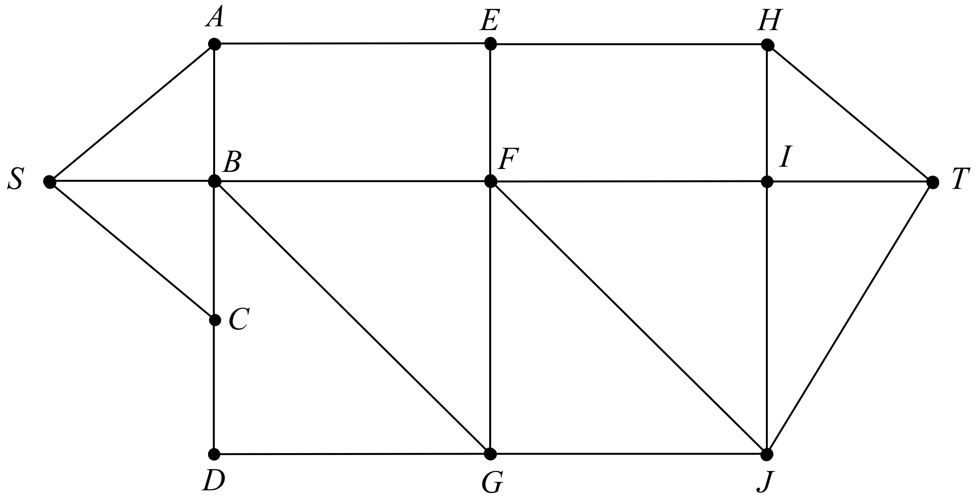
(a) ……………………………………………………………………………………………

(b) (i)



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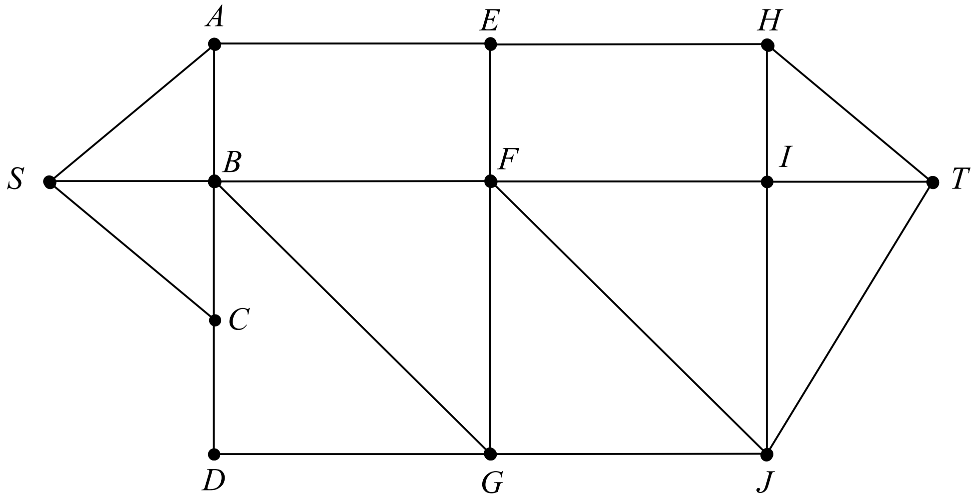
(ii)



Maximum Flow = ……………………………………………………………………………..

(c) ……………………………………………………………………………………………

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(d) ……………………………………………………………………………………………

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(e) ……………………………………………………………………………………………

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