

GCE Examinations
Advanced Subsidiary / Advanced Level
Further Pure Mathematics
Module FP1

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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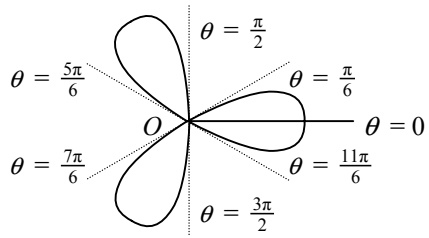
FP1 Paper A – Marking Guide

1. (a) $f(1) = 1 - 5 + 17 - 13 = 0 \therefore (z - 1)$ is a factor B1
- (b) $f(z) = (z - 1)(z^2 - 4z + 13)$ M1 A1
quad. gives $z = \frac{4 \pm \sqrt{16 - 52}}{2}$ M1
 \therefore roots are $1, 2 - 3i, 2 + 3i$ A2 (6)
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2. $\frac{dy}{dx} + \frac{3}{x}y = \frac{e^x}{x^3}$ M1
 \therefore int. fac. = $e^{\int \frac{3}{x} dx} = e^{3 \ln|x|} = x^3$ M1 A1
 $x^3 \frac{dy}{dx} + 3x^2 y = e^x$
 $\frac{d}{dx}(x^3 y) = e^x$ M1
 $x^3 y = \int e^x dx = e^x + c \therefore y = \frac{e^x + c}{x^3}$ M1 A1 (6)
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3. (a) $\frac{1}{r(r+1)} \equiv \frac{A}{r} + \frac{B}{r+1}$ giving $A = 1, B = -1$
 $\therefore \frac{1}{r(r+1)} \equiv \frac{1}{r} - \frac{1}{r+1}$ M1 A1
- (b) $\sum_{r=3}^{35} \left(\frac{1}{r} - \frac{1}{r+1}\right) = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{35}\right) - \left(\frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{35} + \frac{1}{36}\right)$ M1 A1
 $= \frac{1}{3} - \frac{1}{36} = \frac{11}{36}$ M1 A1 (6)
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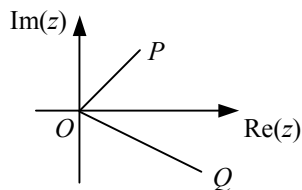
4. $\frac{(x-3)^2}{x+1} - 2 < 0 \therefore \frac{x^2 - 8x + 7}{x+1} < 0$ M1 A1
 $\frac{(x-1)(x-7)}{x+1} < 0 \therefore$ critical values are $-1, 1, 7$ M1 A1
considering change of sign of factors gives
 $x < -1$ or $1 < x < 7$ M1 A2 (7)
-

5. (a)  B3
- (b) area = $6 \times \frac{1}{2} \int_0^{\frac{\pi}{6}} (a \cos 3\theta)^2 d\theta = 3a^2 \int_0^{\frac{\pi}{6}} \cos^2 3\theta d\theta$ M1 A1
 $= \frac{3}{2} a^2 \int_0^{\frac{\pi}{6}} 1 + \cos 6\theta d\theta$ M1
 $= \frac{3}{2} a^2 \left[\theta + \frac{1}{6} \sin 6\theta\right]_0^{\frac{\pi}{6}}$ A1
giving $\frac{1}{4} \pi a^2$ M1 A1 (9)
-

6. (a) 2 solutions B1
- (b) $f(0) = -1$; $f(1) = 1.64$
 f cont. over interval, change of sign \therefore root M1 A1
- (c) $f'(x) = e^x + 2 \sin x$ M1
 $x_{n+1} = x_n - \frac{e^{x_n} - 2 \cos x_n}{e^{x_n} + 2 \sin x_n}$ A1
giving 0.54 (2dp) M1 A1
- (d) $f(0.535) = -0.0131$; $f(0.545) = 0.0144$
change of sign \therefore root \therefore correct to 2dp M1 A1
- (e) $f(-2) = 0.967629$; $f(-1) = -0.712725$
 $\beta \approx -2 + \frac{0.967629}{0.967629 + 0.712725} \times 1$ M1 A1
 $= -1.42$ (2dp) A1 (12)

7. (a) $\frac{A}{1-i} + \frac{B}{2+i} = 6 \therefore A(2+i) + B(1-i) = 6(1-i)(2+i)$ M1
giving $2A + B + i(A - B) = 18 - 6i$ A1
 $\therefore 2A + B = 18$
 $A - B = -6$ M1 A1
giving $A = 4, B = 10$ M1 A1

- (b) $z = \frac{4}{1-i} \times \frac{1+i}{1+i} = 2 + 2i$ M1 A1
 $w = \frac{10}{2+i} \times \frac{2-i}{2-i} = 4 - 2i$ A1



- (c) $z - w = (2 + 2i) - (4 - 2i) = -2 + 4i$ M1
 $|z - w| = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$ M1 A1 (14)

8. (a) $\frac{dx}{dt} = -p \sin t + q \cos t, \frac{d^2x}{dt^2} = -p \cos t - q \sin t$ M1 A1
 $-p \cos t - q \sin t - 4p \sin t + 4q \cos t + 3p \cos t + 3q \sin t = \sin t$ M1
 $2p + 4q = 0$
 $2q - 4p = 1$ A1
giving $p = -\frac{1}{5}, q = \frac{1}{10}$ M1 A1

- (b) aux. eqn. $m^2 + 4m + 3 = 0$ M1
 $(m + 1)(m + 3) = 0 \therefore m = -1, -3$ A1
C.F. $x = Ae^{-t} + Be^{-3t}$ A1
gen. soln. $x = Ae^{-t} + Be^{-3t} - \frac{1}{5} \cos t + \frac{1}{10} \sin t$
 $\therefore \frac{dx}{dt} = -Ae^{-t} - 3Be^{-3t} + \frac{1}{5} \sin t + \frac{1}{10} \cos t$ M1 A1
 $t = 0, x = 1, \frac{dx}{dt} = \frac{1}{2}$ gives $1 = A + B - \frac{1}{5}$ and $\frac{1}{2} = -A - 3B + \frac{1}{10}$ M1
solve simul. $A = 2, B = -\frac{4}{5}$ M1 A1
soln. $x = 2e^{-t} - \frac{4}{5}e^{-3t} - \frac{1}{5} \cos t + \frac{1}{10} \sin t$ A1 (15)

Total (75)

Performance Record – FP1 Paper A

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	complex nos.	1 st order diff. eqn	series	inequality	polar coords	numerical methods	complex nos.	2 nd order diff. eqn	
Marks	6	6	6	7	9	12	14	15	75
Student									