GCE Examinations

Further Pure Mathematics
Module FP1

Advanced Subsidiary / Advanced Level

Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. Find the set of values of x for which

$$|2x^2 - 5x| < x. \qquad \textbf{(6 marks)}$$

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2. *(a)* Sketch the curve C with the polar equation

$$r^2 = a^2 \sin^2 2\theta, \qquad 0 \leq \theta < 2\pi. \qquad \textbf{(3 marks)}$$

(b) Find the exact area of the region enclosed by one loop of the curve C. **(5 marks)**

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3. *(a)* Show that

$$\sum_{r=1}^{n} (r^2 + 1)(r - 1) = \tfrac{1}{12} n(n - 1)(3n^2 + 5n + 8). \qquad \textbf{(6 marks)}$$

(b) Hence evaluate

$$\sum_{r=5}^{25} (r^2 + 1)(r - 1). \qquad \textbf{(3 marks)}$$

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4. *(a)* Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y \cot x = \sin 2x. \qquad \textbf{(6 marks)}$$

(b) Given also that $y = 2$ when $x = \frac{\pi}{6}$, find the exact value of y when $x = \frac{2\pi}{3}$. **(3 marks)**

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5. $$\mathrm{f}(x) \equiv x^3 - \ln(4 - x^2), \quad x \in \mathbb{R}, \quad -2 < x < 2.$$

(a) Show that one root, α, of the equation $\mathrm{f}(x) = 0$ lies in the interval $1.0 < \alpha < 1.1$

(2 marks)

(b) Starting with $x = 1.0$, show that using the Newton-Raphson method twice gives an approximation to α that is correct to 6 decimal places.

(8 marks)

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6. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 7 - \mathrm{i}, \quad z_2 = 1 + \mathrm{i}\sqrt{3}, \quad z_3 = a + \mathrm{i}b,$$

where a and b are rational constants.

Given that the modulus of $z_1 z_3$ is 50,

(a) find the modulus of z_3. **(3 marks)**

Given also that the argument of $\frac{z_2}{z_3}$ is $\frac{7\pi}{12}$,

(b) find the argument of z_3. **(3 marks)**

(c) Find the values of a and b. **(2 marks)**

(d) Show that $\frac{z_1}{z_3} = \frac{1}{5}(4 + 3\mathrm{i})$. **(3 marks)**

(e) Represent z_1, z_3 and $\frac{z_1}{z_3}$ on the same Argand diagram. **(2 marks)**

(f) By considering the modulus and argument of z_1 and z_3, explain why $\frac{z_3}{z_1} = \left(\frac{z_1}{z_3}\right)^*$.

(2 marks)

Turn over

7. *(a)* Given that $x = \mathrm{e}^t$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}t}$ and show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{e}^{-2t}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t}\right).$$ **(5 marks)**

(b) Show that the substitution $x = \mathrm{e}^t$ transforms the differential equation

$$x^2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 6x^2$$

into the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 2\frac{\mathrm{d}y}{\mathrm{d}t} - 3y = 6\mathrm{e}^{2t}.$$ **(3 marks)**

(c) Given that when $x = 1$, $y = 3$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -5$, solve the differential equation

$$x^2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 6x^2.$$ **(10 marks)**

END