GCE Examinations

Further Pure Mathematics Module FP1

Advanced Subsidiary / Advanced Level

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. Find the set of values of x for which

$$|x-2| > 2|x+1|.$$ **(6 marks)**

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2. *(a)* By using the substitution $y = vx$, or otherwise, find the general solution of the differential equation

$$xy\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y^2.$$ **(7 marks)**

(b) Given also that $y = 2$ when $x = 1$, show that for $x > 0$

$$y^2 = 2x^2(\ln x + 2).$$ **(2 marks)**

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3. *(a)* Find the sum of the series

$$2^3 + 4^3 + 6^3 + \ldots + (2n)^3,$$

giving your answer in a simplified form. **(3 marks)**

(b) Hence, or otherwise, show that the sum of the series

$$1^3 - 2^3 + 3^3 - 4^3 + \ldots + (2n-1)^3 - (2n)^3$$

is $-n^2(4n + 3)$. **(6 marks)**

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4. Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = 2\mathrm{e}^{3x}.$$ **(10 marks)**

5.

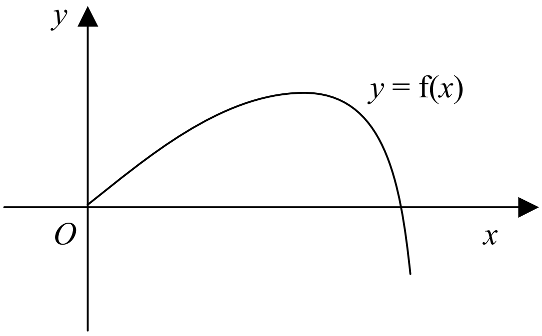

Fig. 1

Figure 1 shows part of the curve $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) \equiv 2x - \tan x,\ \ x \in \mathbb{R},\ \ 0 \leq x < \tfrac{\pi}{2}\ .$$

(a) Show that there is a root, α, of the equation $\mathrm{f}(x) = 0$ in the interval $(1, 1.5)$. **(2 marks)**

(b) Use the Newton-Raphson method with an initial value of $x = 1.25$ to find α correct to 2 decimal places and justify the accuracy of your answer.
(7 marks)

(c) Explain with the aid of a diagram why the Newton-Raphson method fails if an initial value of $x = 0.75$ is used when trying to find α.
(3 marks)

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6. The complex numbers z and w are defined such that

$$3z + w = 14,\ \text{ and}$$
$$z - \mathrm{i}w = 15 - 9\mathrm{i}.$$

(a) Show that $z = 3 - 4\mathrm{i}$ and find w in the form $a + \mathrm{i}b$, where a and b are real numbers.

(6 marks)

(b) Find the square roots of z in the form $c + \mathrm{i}d$, where c and d are real numbers.
(7 marks)

Turn over

7.

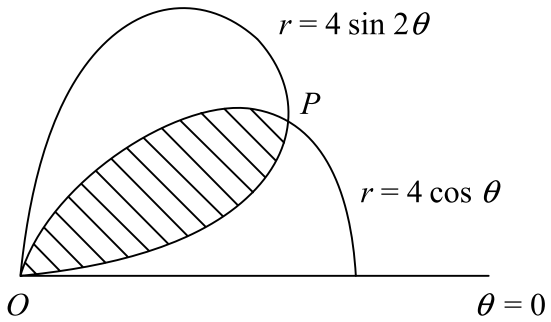

Fig. 2

Figure 2 shows the curves with polar equations

$$r = 4\sin 2\theta \quad 0 \le \theta \le \tfrac{\pi}{2},$$

$$r = 4\cos\theta \quad 0 \le \theta \le \tfrac{\pi}{2}.$$

(a) Find the polar coordinates of the point P where the two curves intersect. **(5 marks)**

(b) Find the exact area of the shaded region bounded by the two curves. **(11 marks)**

END