GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP1

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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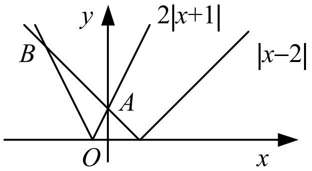
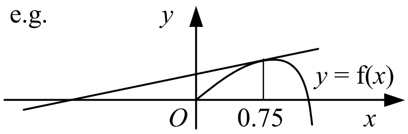
FP1 Paper C – Marking Guide

1.



1.	(graph)	B2	
	by inspection, at A $x = 0$	B1	
	at B $-(x - 2) = -2(x + 1)$ giving $x = -4$	M1 A1	
	using graphs, require $-4 < x < 0$	A1	**(6)**
2.	*(a)* $\frac{dy}{dx} = v + x\frac{dv}{dx}$	M1 A1	
	$x.vx.(v + x\frac{dv}{dx}) = x^2 + (vx)^2 \quad \therefore\ xv\frac{dv}{dx} = 1$	M1 A1	
	$\int v \, dv = \int \frac{1}{x} \, dx$	M1	
	$\frac{1}{2}v^2 = \ln\lvert x \rvert + c$	A1	
	giving e.g. $y^2 = x^2(k + \ln x^2)$	A1	
	(b) $x = 1, y = 2 \quad \therefore\ k = 4$	M1	
	$x > 0 \ \therefore\ y^2 = x^2(4 + 2\ln x) = 2x^2(\ln x + 2)$	A1	**(9)**
3.	*(a)* series sum $= \sum_{r=1}^{n} (2r)^3 = 8 \times \sum_{r=1}^{n} r^3 = 8 \times \frac{1}{4}n^2(n + 1)^2 = 2n^2(n + 1)^2$	M2 A1	
	(b) series sum $= \sum_{r=1}^{2n} r^3 - 2 \times \sum_{r=1}^{n} (2r)^3$	M1 A1	
	$= \frac{1}{4}(2n)^2(2n + 1)^2 - 2 \times 2n^2(n + 1)^2$	M1	
	$= n^2[(2n + 1)^2 - 4(n + 1)^2]$	A1	
	giving $-n^2(4n + 3)$	M1 A1	**(9)**
4.	aux. eqn. $m^2 - 6m + 9 = 0$	M1	
	$(m - 3)^2 = 0 \quad \therefore\ m = 3$	A1	
	C.F. $y = (A + Bx)e^{3x}$	A1	
	P.I. try $y = Cx^2e^{3x}$	M1	
	$\frac{dy}{dx} = 3Cx^2e^{3x} + 2Cxe^{3x}, \quad \frac{d^2y}{dx^2} = 9Cx^2e^{3x} + 12Cxe^{3x} + 2Ce^{3x}$	M1 A2	
	$\therefore\ 9Cx^2e^{3x} + 12Cxe^{3x} + 2Ce^{3x} - 18Cx^2e^{3x} - 12Cxe^{3x} + 9Cx^2e^{3x} = 2e^{3x}$	M1	
	giving $2Ce^{3x} = 2e^{3x} \quad \therefore\ C = 1$	A1	
	gen. soln. $y = (A + Bx)e^{3x} + x^2e^{3x} = e^{3x}(A + Bx + x^2)$	A1	**(10)**

5.	*(a)*	$f(1) = 0.443$; $f(1.5) = -11.1$		
		f cont. over interval, change of sign $\therefore$ root	M1 A1	
	(b)	$f'(x) = 2 - \sec^2 x$	M1	
		$x_{n+1} = x_n - \frac{2x_n - \tan x_n}{2 - \sec^2 x_n}$	A1	
		$x_0 = 1.25$, $x_1 = 1.1868$, $x_2 = 1.1670$, $x_3 = 1.1656$, $x_4 = 1.1656$	M1 A1	
		$\alpha = 1.17$ (2dp)	A1	
		$f(1.165) = 0.00248$; $f(1.175) = -0.0432$		
		change of sign $\therefore$ root $\therefore$ accurate to 2dp	M1 A1	
	(c)	e.g. 	B1	
		N-R uses intersec. of tangent at initial value with x-axis as next approx.	B1	
		at $x = 0.75$ this tangent is further away from root	B1	**(12)**
6.	*(a)*	$z = 15 - 9i + iw$ $\therefore$ $45 - 27i + 3iw + w = 14$	M1	
		$w(1 + 3i) = -31 + 27i$	M1	
		$\therefore w = \frac{-31+27i}{1+3i} \times \frac{1-3i}{1-3i} = \frac{50+120i}{10} = 5 + 12i$	M1 A1	
		$3z = 14 - (5 + 12i) = 9 - 12i$ $\therefore z = 3 - 4i$	M1 A1	
	(b)	$(p + iq)^2 = 3 - 4i$; p, q real	M1	
		$p^2 + 2pqi - q^2 = 3 - 4i$	M1	
		$\therefore p^2 - q^2 = 3$; $2pq = -4$	A1	
		e.g. sub. for q giving $(p^2 - 4)(p^2 + 1) = 0$	M1 A1	
		$p^2 = -1$ (no solns) or 4 $\therefore p = \pm 2$	M1	
		sub. in giving $(2 - i)$ and $(-2 + i)$	A1	**(13)**
7.	*(a)*	$4 \sin 2\theta = 4 \cos \theta$		
		$4 \cos \theta(2 \sin \theta - 1) = 0$	M1	
		$\sin \theta = \frac{1}{2}$ or $\cos \theta = 0$ giving $\theta = \frac{\pi}{6}$ or $\frac{\pi}{2}$	M1 A1	
		at P, $\theta = \frac{\pi}{6}$ $\therefore P$ is $(2\sqrt{3}, \frac{\pi}{6})$	M1 A1	
	(b)	area $= \frac{1}{2}\int_0^{\frac{\pi}{6}} (4 \sin 2\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4 \cos \theta)^2 \, d\theta$	M1 A1	
		$= 4\int_0^{\frac{\pi}{6}} 1 - \cos 4\theta \, d\theta + 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 + \cos 2\theta \, d\theta$	M1 A2	
		$= 4[\theta - \frac{1}{4}\sin 4\theta]_0^{\frac{\pi}{6}} + 4[\theta + \frac{1}{2}\sin 2\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$	M1 A2	
		$= 4[(\frac{\pi}{6} - \frac{\sqrt{3}}{8}) - (0) + (\frac{\pi}{2}) - (\frac{\pi}{6} + \frac{\sqrt{3}}{4})]$	M1 A1	
		$= 4[\frac{\pi}{2} - \frac{3\sqrt{3}}{8}] = 2\pi - \frac{3}{2}\sqrt{3}$	A1	**(16)**
			Total	**(75)**

Performance Record – FP1 Paper C

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	inequality	1st order diff. eqn	series	2nd order diff. eqn	numerical methods	complex nos.	polar coords	
Marks	6	9	9	10	12	13	16	75
Student								