

GCE Examinations

Further Pure Mathematics

Module FP1

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



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1. The function f is defined by

$$f(x) \equiv 3x^3 + kx^2 + 42x + k,$$

where k is an integer.

Given that $(3 + i)$ is a root of the equation $f(x) = 0$,

- (a) find a quadratic factor of $f(x)$, **(3 marks)**
(b) find the value of k . **(4 marks)**
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2. Find the set of values of x for which

$$\frac{x}{x-1} > \frac{2}{3-x}. \quad \text{(8 marks)}$$

3. Given that $y = \frac{1}{2}$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} - 3x + 4xy = 0,$$

giving your answer in the form $y = f(x)$. **(8 marks)**

4. (a) Express $\frac{3r+4}{r(r+1)(r+2)}$ in partial fractions. **(3 marks)**

(b) Hence, show that

$$\sum_{r=1}^n \frac{3r+4}{r(r+1)(r+2)} = \frac{n(5n+9)}{2(n+1)(n+2)}. \quad \text{(7 marks)}$$

5. (a) Find the values of a , b and c such that $y = ax^2 + bx + c$ satisfies the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 5x^2 - 13x + 1. \quad (5 \text{ marks})$$

- (b) Hence, find the general solution of this differential equation. (5 marks)
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6. $f(x) \equiv \frac{2}{3}x + \sin 2x - 1, \quad x \in \mathbb{R}.$

- (a) By sketching the graphs of $y = \sin 2x$ and $y = 1 - \frac{2}{3}x$ on the same diagram, find the number of solutions to the equation $f(x) = 0$. (3 marks)

- (b) (i) Show that one root, α , of the equation $f(x) = 0$ lies in the interval $(2.5, 3)$.
- (ii) Use one application of the method of linear interpolation on this interval to find an approximate value for α , giving your answer correct to 2 decimal places.
- (iii) Determine whether or not your answer to part (ii) gives the value of α correct to 2 decimal places.

(7 marks)

- (c) Use the Newton-Raphson method with a starting value of $x = 0.5$ to find another root of the equation $f(x) = 0$ correct to 3 significant figures.

(5 marks)

Turn over

7.

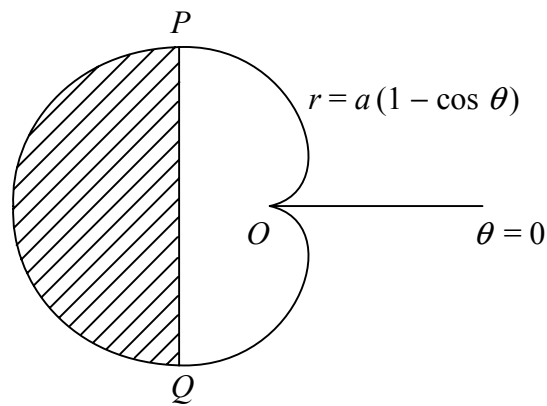


Fig. 1

Figure 1 shows the curve C with polar equation

$$r = a(1 - \cos \theta), \quad 0 \leq \theta < 2\pi,$$

where a is a positive constant.

At the points P and Q the tangents to the curve are parallel to the initial line $\theta = 0$.

(a) Find the polar coordinates of P and Q . (7 marks)

The shaded region is bounded by the curve C and the straight line PQ .

(b) Show that the area of the shaded region is $\frac{1}{16}a^2(8\pi + 9\sqrt{3})$. (10 marks)

END