GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics

Module FP1

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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FP1 Paper D – Marking Guide

1. *(a)* (3 – i) must also be root — B1

$\therefore [x - (3 + i)][x - (3 - i)] = x^2 - 6x + 10$ is a factor — M1 A1

(b) linear factor is $(3x + \frac{k}{10})$ — M1 A1

$\therefore$ coeff. of $x = 30 - \frac{6k}{10} = 42$ — M1

giving $k = -20$ — A1 **(7)**

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2. $\frac{x}{x-1} - \frac{2}{3-x} > 0 \quad \therefore \quad \frac{x^2-x-2}{(x-1)(x-3)} > 0$ — M1 A1

$\frac{(x-2)(x+1)}{(x-1)(x-3)} > 0 \quad \therefore$ critical values are $-1, 1, 2, 3$ — M1 A1

considering change of sign of factors gives

$x < -1$ or $1 < x < 2$ or $x > 3$ — M2 A2 **(8)**

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3. $\frac{dy}{dx} + 4xy = 3x \quad \therefore$ int. fac. $= e^{\int 4x\,dx} = e^{2x^2}$ — M1 A1

$e^{2x^2}\frac{dy}{dx} + 4xy\,e^{2x^2} = 3x\,e^{2x^2}$

$\frac{d}{dx}(y\,e^{2x^2}) = 3x\,e^{2x^2}$ — M1

$y\,e^{2x^2} = \int 3x\,e^{2x^2}\,dx = \frac{3}{4}e^{2x^2} + c$ — M1 A1

$x = 0, y = \frac{1}{2} \quad \therefore c = -\frac{1}{4}$ — M1 A1

$\therefore y = \frac{1}{4}(3 - e^{-2x^2})$ — A1 **(8)**

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4. *(a)* $\frac{3r+4}{r(r+1)(r+2)} \equiv \frac{A}{r} + \frac{B}{r+1} + \frac{C}{r+2}$ — M1

giving $A = 2, B = -1, C = -1 \quad \therefore \frac{3r+4}{r(r+1)(r+2)} \equiv \frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2}$ — M1 A1

(b) $\sum_{r=1}^{n}(\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2})$

$= (\frac{2}{1} + \frac{2}{2} + \frac{2}{3} + \ldots + \frac{2}{n}) - (\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} + \frac{1}{n+1}) - (\frac{1}{3} + \ldots + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2})$ — M1 A2

$= \frac{2}{1} + \frac{2}{2} - \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+1} - \frac{1}{n+2}$ — M1

$= \frac{5}{2} - \frac{2}{n+1} - \frac{1}{n+2}$ — A1

giving $\frac{n(5n+9)}{2(n+1)(n+2)}$ — M1 A1 **(10)**

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5. *(a)* $\frac{dy}{dx} = 2ax + b \quad \frac{d^2y}{dx^2} = 2a$ — M1 A1

$2a + 4ax + 2b + 10ax^2 + 10bx + 10c = 5x^2 - 13x + 1$ — M1

giving $a = \frac{1}{2}, b = -\frac{3}{2}, c = \frac{3}{10}$ — M1 A1

(b) aux. eqn. $m^2 + 2m + 10 = 0$ — M1

$m = -1 \pm 3i$ — M1 A1

gen. soln. $y = e^{-x}(A\cos 3x + B\sin 3x) + \frac{1}{2}x^2 - \frac{3}{2}x + \frac{3}{10}$ — A2 **(10)**

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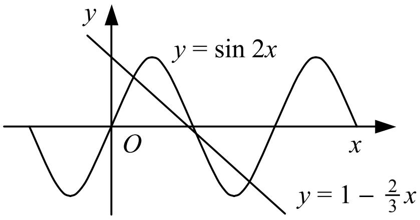
6. *(a)*



B2

3 pts of intersec. $\therefore$ 3 solutions — B1

(b) (i) $f(2.5) = -0.292$; $f(3) = 0.721$

f cont. over interval, change of sign $\therefore$ root — M1 A1

(ii) $\alpha \approx 2.5 + \frac{0.292258}{0.292258+0.720585} \times 0.5 = 2.64$ (2dp) — M1 A2

(iii) $f(2.635) = -0.0919$; $f(2.645) = -0.0744$

no change of sign $\therefore$ no root $\therefore$ not correct to 2dp — M1 A1

(c) $f'(x) = \frac{2}{3} + 2\cos 2x$ — M1

$x_{n+1} = x_n - \dfrac{\frac{2}{3}x_n + \sin 2x_n - 1}{\frac{2}{3} + 2\cos 2x_n}$ — A1

$x_1 = 0.399956$, $x_2 = 0.407755$, $x_3 = 0.407798$ — M1 A1

root = 0.408 (3sf) — A1 **(15)**

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7. *(a)* require $\dfrac{d(r\sin\theta)}{d\theta} = 0$ — M1

$r\sin\theta = a\sin\theta(1-\cos\theta) = a(\sin\theta - \frac{1}{2}\sin 2\theta)$ — M1

$\therefore \dfrac{d(r\sin\theta)}{d\theta} = a(\cos\theta - \cos 2\theta)$ — A1

giving $2\cos^2\theta - \cos\theta - 1 = (2\cos\theta + 1)(\cos\theta - 1) = 0$ — M1

$\therefore \cos\theta = -\frac{1}{2}, 1$ so $\theta = (0), \frac{2\pi}{3}, \frac{4\pi}{3}$ — M1 A1

$\therefore P(\frac{3}{2}a, \frac{2\pi}{3})$, $Q(\frac{3}{2}a, \frac{4\pi}{3})$ — A1

(b) area bounded by *C* and lines *OP* and *OQ*

$= \frac{1}{2}\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} a^2(1-\cos\theta)^2 \, d\theta = \frac{1}{2}a^2\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 1 - 2\cos\theta + \cos^2\theta \, d\theta$ — M1 A1

giving $\frac{1}{4}a^2\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 3 - 4\cos\theta + \cos 2\theta \, d\theta$ — M1 A1

$= \frac{1}{4}a^2[3\theta - 4\sin\theta + \frac{1}{2}\sin 2\theta]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}$ — A1

giving $\frac{1}{8}a^2(4\pi + 9\sqrt{3})$ — M1 A1

area of triangle $OPQ = \frac{1}{2} \times \frac{3}{2}a \times \frac{3}{2}a \times \sin\frac{2\pi}{3} = \frac{9}{16}a^2\sqrt{3}$ — M1 A1

shaded area $= \frac{1}{8}a^2(4\pi + 9\sqrt{3}) - \frac{9}{16}a^2\sqrt{3} = \frac{1}{16}a^2(8\pi + 9\sqrt{3})$ — A1 **(17)**

Total **(75)**

Performance Record – FP1 Paper D

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	complex nos.	inequality	1^{st} order diff. eqn	series	2^{nd} order diff. eqn	numerical methods	polar coords	
Marks	7	8	8	10	10	15	17	75
Student								