GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP1

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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FP1 Paper E – Marking Guide

1. *(a)* $w = \frac{10+5i}{2-i} \times \frac{2+i}{2+i} = \frac{15+20i}{5} = 3 + 4i$ M1 A2

(b) let $z = x + iy$ $\therefore x + iy + 2(x - iy) = 3 + 4i$ M1

$\therefore 3x = 3, -y = 4$ M1 A1

$x = 1, y = -4$ $\therefore z = 1 - 4i$ A1 **(7)**

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2. $\sum_{r=0}^{n} (r+1)(r+2) = \sum_{r=1}^{n+1} r(r+1) = \sum_{r=1}^{n+1} (r^2 + r)$ M1 A2

$= \frac{1}{6}(n+1)(n+2)(2n+3) + \frac{1}{2}(n+1)(n+2)$ M1 A1

$= \frac{1}{6}(n+1)(n+2)[(2n+3)+3]$ M1

$= \frac{1}{6}(n+1)(n+2)[2n+6]$

$= \frac{1}{3}(n+1)(n+2)(n+3)$ A1 **(7)**

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3. $\frac{dy}{dx} - y = x$ $\therefore$ int. fac. $= e^{\int -1\,dx} = e^{-x}$ M1 A1

$\therefore e^{-x}\frac{dy}{dx} - ye^{-x} = xe^{-x}$ M1

$\frac{d}{dx}(ye^{-x}) = xe^{-x}$

$ye^{-x} = \int xe^{-x}\,dx$ A1

$u = x, u' = 1; v' = e^{-x}, v = -e^{-x}$ M1

$ye^{-x} = -xe^{-x} - \int -e^{-x}\,dx$ A1

$ye^{-x} = -xe^{-x} - e^{-x} + c$ A1

$y = ce^{x} - x - 1$

$x = 0, y = 0$ $\therefore c = 1$ M1

$\therefore y = e^{x} - x - 1$ A1 **(9)**

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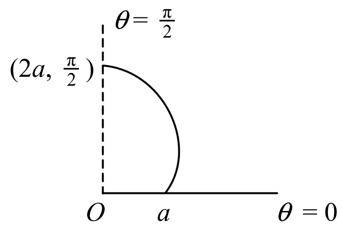
4. *(a)*



B2

(b) require $\frac{d(r\cos\theta)}{d\theta} = 0$ M1

$r\cos\theta = a\cos\theta(1 + \sin\theta) = a(\cos\theta + \cos\theta\sin\theta)$ A1

$\therefore \frac{d(r\cos\theta)}{d\theta} = a[-\sin\theta + \cos\theta(\cos\theta) + \sin\theta(-\sin\theta)]$ M1

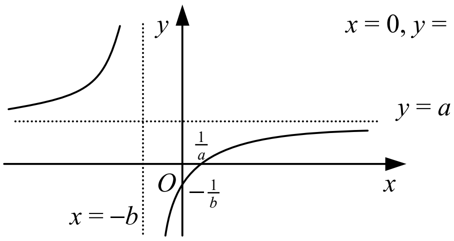
$\therefore -\sin\theta + (1 - \sin^2\theta) - \sin^2\theta = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$ A1

$(2\sin\theta - 1)(\sin\theta + 1) = 0$ M1

$\therefore \sin\theta = \frac{1}{2}, -1$ A1

$0 \le \theta \le \frac{\pi}{2}$ $\therefore \theta = \frac{\pi}{6}$ giving $(\frac{3}{2}a, \frac{\pi}{6})$ M1 A1 **(10)**

5.	*(a)*	$y = \dfrac{ax+ab-ab-1}{x+b} = a - \dfrac{ab+1}{x+b}$ $\therefore$ asymptotes are $x = -b$ and $y = a$	M1 A2	
	(b)	$x = 0,\ y = -\frac{1}{b}$; $y = 0,\ x = \frac{1}{a}$	B1	
			B2	
	(c)	$3x - 1 = 2(x + 2)$ gives $x = 5$	M1 A1	
		$-(3x - 1) = 2(x + 2)$ gives $x = -\frac{3}{5}$	M1 A1	
		considering curve below x-axis reflected in x-axis gives $-\frac{3}{5} < x < 5$	M1 A1	**(12)**
6.	*(a)*	let $f(x) = e^x - 4\sin x$, $f(0) = 1$; $f(1) = -0.648$; f cont. over interval, change of sign $\therefore$ root in interval [0, 1]	M1 A1	
		$f(1) = -0.648$; $f(1.5) = 0.492$ f cont. over interval, change of sign $\therefore$ root in interval [1, 1.5]	A1	
	(b)	$f'(x) = e^x - 4\cos x$, $x_{n+1} = x_n - \dfrac{e^{x_n} - 4\sin x_n}{e^{x_n} - 4\cos x_n}$	M1 A2	
		giving $\alpha = 0.37$ (2dp)	M1 A1	
	(c)	$\beta \approx 1 + \dfrac{0.64760}{0.64760+0.49171} \times 0.5 = 1.284... = 1.3$ (1dp)	M1 A2	
	(d)	$f(1.25) = -0.306$; $f(1.35) = -0.0455$ no change of sign $\therefore$ no root $\therefore$ not correct to 1dp	M1 A1	**(13)**
7.	*(a)*	(i) $\dfrac{dx}{dt} = \frac{1}{2} t^{-\frac{1}{2}}$ $\therefore$ $\dfrac{dt}{dx} = 2t^{\frac{1}{2}}$	M1	
		$\dfrac{dy}{dx} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = 2t^{\frac{1}{2}}\dfrac{dy}{dt}$	M1 A1	
		(ii) $\dfrac{d^2y}{dx^2} = \dfrac{dt}{dx}(2t^{\frac{1}{2}}\dfrac{d^2y}{dt^2} + t^{-\frac{1}{2}}\dfrac{dy}{dt})$	M1 A1	
		$= 2t^{\frac{1}{2}}(2t^{\frac{1}{2}}\dfrac{d^2y}{dt^2} + t^{-\frac{1}{2}}\dfrac{dy}{dt}) = 2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2}$	A1	
	(b)	$\dfrac{1}{t}(2\dfrac{dy}{dt} + 4t\dfrac{d^2y}{dt^2}) + (4t^{-\frac{1}{2}} - t^{-\frac{3}{2}})2t^{\frac{1}{2}}\dfrac{dy}{dt} + 3y = 3t + 5$	M1 A1	
		$2t^{-1}\dfrac{dy}{dt} + 4\dfrac{d^2y}{dt^2} + 8\dfrac{dy}{dt} - 2t^{-1}\dfrac{dy}{dt} + 3y = 3t + 5$	M1	
		giving $4\dfrac{d^2y}{dt^2} + 8\dfrac{dy}{dt} + 3y = 3t + 5$	A1	
	(c)	aux. eqn. $4m^2 + 8m + 3 = 0$	M1	
		$(2m + 1)(2m + 3) = 0$; $m = -\frac{1}{2}, -\frac{3}{2}$ C.F. $y = Ae^{-\frac{1}{2}t} + Be^{-\frac{3}{2}t}$	A1	
		for P.I. try $y = at + b$ $\therefore$ $\dfrac{dy}{dt} = a$, $\dfrac{d^2y}{dt^2} = 0$	M1	
		so $4(0) + 8a + 3(at + b) = 3t + 5$	M1	
		$\therefore$ $3a = 3$; $8a + 3b = 5$ giving $a = 1, b = -1$	A1	
		gen. soln. $y = Ae^{-\frac{1}{2}t} + Be^{-\frac{3}{2}t} + t - 1$	M1	
		$\therefore$ gen. soln. of (I): $y = Ae^{-\frac{1}{2}x^2} + Be^{-\frac{3}{2}x^2} + x^2 - 1$	A1	**(17)**
			Total	**(75)**

Performance Record – FP1 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	complex nos.	series	1^{st} order diff. eqn	polar coords	inequality	numerical methods	2^{nd} order diff. eqn	
Marks	7	7	9	10	12	13	17	75
Student								