GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP1

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

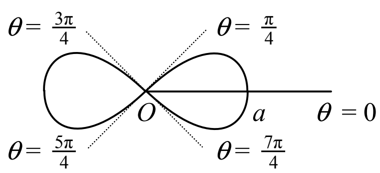
Method marks (M) are awarded for knowing and using a method.

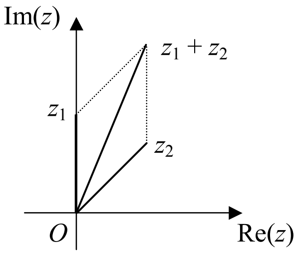
Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Shaun Armstrong & Chris Huffer

FP1 Paper F – Marking Guide

1.		area = $\frac{1}{2}\int_0^{2\pi} a^2\theta^2 \; \mathrm{d}\theta$	M1	
		$= \frac{1}{2}a^2[\frac{1}{3}\theta^3]_0^{2\pi}$	A1	
		giving $\frac{4}{3}a^2\pi^3$	M1 A1	**(4)**
2.		$\frac{(x-1)(x+2)}{x+4} - 4 > 0 \;\therefore\; \frac{x^2-3x-18}{x+4} > 0$	M1 A1	
		$\frac{(x+3)(x-6)}{x+4} > 0 \;\therefore$ critical values are $-4, -3, 6$	M1 A1	
		considering change of sign of factors gives $-4 < x < -3$ or $x > 6$	M1 A2	**(7)**
3.	*(a)*	$\mathrm{f}(0) = 3$; $\mathrm{f}(1) = -1$; f cont. over interval, change of sign $\therefore$ root	M1 A1	
	(b)	$\alpha \approx 0 + \frac{3}{3+1} \times 1 = 0.75$	M1 A1	
	(c)	$\mathrm{f}'(x) = 15x^4 - 14x,\; x_{n+1} = x_n - \dfrac{3x_n^5 - 7x_n^2 + 3}{15x_n^4 - 14x_n}$	M1 A1	
		giving $\beta \approx -0.623$ (3dp)	A1	**(7)**
4.	*(a)*	$x = r\cos\theta,\; y = r\sin\theta$		
		$\therefore\; (r^2\cos^2\theta + r^2\sin^2\theta)^2 = a^2(r^2\cos^2\theta - r^2\sin^2\theta)$	M1 A1	
		$[r^2(\cos^2\theta + \sin^2\theta)]^2 = a^2r^2(\cos^2\theta - \sin^2\theta)$	M1	
		$r^4 = a^2r^2\cos 2\theta$ giving $r^2 = a^2\cos 2\theta$	A1	
	(b)		B3	**(7)**
5.	*(a)*	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{u^2}\frac{\mathrm{d}u}{\mathrm{d}x}$	M1	
		$\therefore\; -\frac{1}{u^2}\frac{\mathrm{d}u}{\mathrm{d}x} + \frac{1}{xu} - \frac{x}{u^2} = 0$	M1	
		$\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{u}{x} + x = 0$	A1	
	(b)	int. fac. $= \mathrm{e}^{\int -\frac{1}{x}\,\mathrm{d}x} = \mathrm{e}^{-\ln\lvert x\rvert} = \frac{1}{x}$	M1 A1	
		$\therefore\; \frac{1}{x}\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{u}{x^2} = -1$		
		$\frac{\mathrm{d}}{\mathrm{d}x}(\frac{u}{x}) = -1$	M1	
		$\frac{u}{x} = \int -1 \;\mathrm{d}x = -x + c$	A1	
		$\therefore\; \frac{1}{xy} = -x + c$	A1	
		$x = 1, y = 1 \;\therefore\; c = 2$	M1	
		$\therefore\; \frac{1}{xy} = 2 - x$ giving $y = \frac{1}{x(2-x)}$	A1	**(10)**

6.	*(a)*	$\sum_{r=n+1}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n} r^2$	M1	
		$= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}n(n+1)(2n+1)$	A1	
		$= \frac{1}{6}n(2n+1)[2(4n+1) - (n+1)]$	M1	
		$= \frac{1}{6}n(2n+1)(7n+1)$	A1	
	(b)	$\text{LHS} = \dfrac{\frac{1}{6}n(2n+1)(7n+1)}{\frac{1}{6}n(n+1)(2n+1)} = \dfrac{7n+1}{n+1}$	M1 A1	
		$= \dfrac{7n+7-6}{n+1} = 7 - \dfrac{6}{n+1}$	M1	
		$n \geq 1;\ n+1 \geq 2\ \therefore 0 < \dfrac{6}{n+1} \leq 3\ \therefore 4 \leq 7 - \dfrac{6}{n+1} < 7$ as required	M1 A2	**(10)**
7.	*(a)*	aux. eqn. $2m^2 - 5m - 3 = 0$	M1	
		$(2m+1)(m-3) = 0;\ m = -\frac{1}{2}, 3$ C.F. $x = A\,\mathrm{e}^{-\frac{1}{2}t} + B\mathrm{e}^{3t}$	A1	
		for P.I. try $x = p\sin t + q\cos t$	M1	
		$\dfrac{\mathrm{d}x}{\mathrm{d}t} = p\cos t - q\sin t,\ \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -p\sin t - q\cos t$	M1 A1	
		$-2p\sin t - 2q\cos t - 5p\cos t + 5q\sin t - 3p\sin t - 3q\cos t = 20\sin t$	M1	
		$-5p + 5q = 20$		
		$-5p - 5q = 0$	A1	
		giving $p = -2, q = 2$	M1 A1	
		$\therefore\ x = A\,\mathrm{e}^{-\frac{1}{2}t} + B\mathrm{e}^{3t} - 2\sin t + 2\cos t$	A1	
	(b)	x finite as $t \to \infty\ \therefore B = 0$	M1 A1	
		$t = 0, x = 5\ \therefore\ 5 = A - 0 + 2$ giving $A = 3$	M1	
		$\therefore\ x = 3\,\mathrm{e}^{-\frac{1}{2}t} - 2\sin t + 2\cos t$	A1	**(14)**
8.	*(a)*	$z_1 = \frac{1+\mathrm{i}}{1-\mathrm{i}} \times \frac{1+\mathrm{i}}{1+\mathrm{i}} = \frac{2\mathrm{i}}{2} = \mathrm{i}$	M1 A1	
	(b)	mod $z_1 = 1$; arg $z_1 = \frac{\pi}{2}$	B2	
	(c)	$z_2 = \frac{\sqrt{2}}{1-\mathrm{i}} \times \frac{1+\mathrm{i}}{1+\mathrm{i}} = \frac{\sqrt{2}}{2}(1+\mathrm{i})$	M1 A1	
		mod $z_2 = \frac{\sqrt{2}}{2} \times \sqrt{2} = 1$	A1	
		arg $z_2 = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$	A1	
	(d)		B3	
		arg $(z_1 + z_2) = \frac{1}{2}(\frac{\pi}{2} + \frac{\pi}{4}) = \frac{3\pi}{8}$	M1 A1	
		$z_1 + z_2 = \mathrm{i} + \frac{\sqrt{2}}{2}(1+\mathrm{i}) = \frac{\sqrt{2}}{2} + (1 + \frac{\sqrt{2}}{2})\mathrm{i}$	B1	
		$\therefore \tan\frac{3\pi}{8} = \dfrac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \dfrac{2+\sqrt{2}}{\sqrt{2}} = \sqrt{2} + 1$	M1 A1	**(16)**

Total **(75)**

Performance Record – FP1 Paper F

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	polar coords	inequality	numerical methods	polar coords	1st order diff. eqn	series	2nd order diff. eqn	complex nos.	
Marks	4	7	7	7	10	10	14	16	75
Student									