GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP1

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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FP1 Paper H – Marking Guide

1.	*(a)*	$f(r+1) - f(r) = (r+1)! - r!$		
		$= r![(r+1) - 1] = r \times r!$	M1 A1	
	(b)	$\sum_{r=1}^{n} (r \times r!) = \sum_{r=1}^{n} [(r+1)! - r!]$	M1	
		$= [2! + 3! + \ldots + n! + (n+1)!] - [1! + 2! + 3! + \ldots + n!]$	M1 A1	
		$= (n+1)! - 1! = (n+1)! - 1$	A1	**(6)**
2.	*(a)*	$y(x^2 + 9) = 2x \;\therefore\; yx^2 - 2x + 9y = 0$	M1	
		$x^2 - \frac{2}{y}x + 9 = 0 \;\therefore\; (x - \frac{1}{y})^2 - \frac{1}{y^2} + 9 = 0$	M1 A1	
		giving $x = \frac{1}{y} \pm \sqrt{\frac{1}{y^2} - 9}$	M1 A1	
	(b)	for x to be real, $\frac{1}{y^2} - 9 \geq 0$	M1	
		$y^2 \geq 0 \;\therefore\; 1 \geq 9y^2$ so $y^2 \leq \frac{1}{9}$	A1	
		$\therefore\; -\frac{1}{3} \leq y \leq \frac{1}{3}$ as required with $a = 3$	A1	**(8)**
3.		$x\frac{dy}{dx} + y(x+1) = 1;\quad \frac{dy}{dx} + (\frac{x+1}{x})y = \frac{1}{x}$	M1 A1	
		int. fac. $= e^{\int 1 + \frac{1}{x}\,dx} = e^{x + \ln x} = e^x e^{\ln x} = xe^x$	M1 A2	
		$\therefore\; xe^x\frac{dy}{dx} + y(x+1)e^x = e^x$	M1	
		$\frac{d}{dx}(yxe^x) = e^x$	A1	
		$xye^x = \int e^x\,dx = e^x + c$	M1	
		$\therefore\; y = \frac{1}{x} + \frac{c}{xe^x}$	A1	**(9)**
4.	*(a)*	let $f(x) = x^2 - \frac{3}{3x-2}$		
		$f(1) = -2$, $f(2) = 3.25$, $f(1.5) = 1.05$, $f(1.3) = 0.111$, $f(1.2) = -0.435$	M1 A1	
		f cont. over interval, change of sign $\therefore$ root	M1	
		$\therefore\; 1.2 < \alpha < 1.3$ so $N = 12$	A1	
	(b)	$f(1.25) = -0.152$, $\therefore\; 1.25 < \alpha < 1.3$	M1 A1	
		$f(1.275) = -0.0182 \;\therefore\; 1.275 < \alpha < 1.3$	A1	
		$f(1.2875) = 0.0469 \;\therefore\; 1.275 < \alpha < 1.2875$		
		$f(1.28125) = 0.0145 \;\therefore\; 1.275 < \alpha < 1.28125 \;\therefore\; \alpha = 1.28$ (2dp)	M1 A1	**(9)**
5.	*(a)*	$f(i) = 0 \;\therefore\; i^4 - 4i^3 + ki^2 - 4i + 13 = 0$	M1	
		$1 + 4i - k - 4i + 13 = 0$	M1	
		$\therefore\; 14 - k = 0$ so $k = 14$	A1	
	(b)	i is a root $\therefore$ –i is a root	B1	
		$\therefore\; (z - i)(z + i) = (z^2 + 1)$ is a factor	M1 A1	
		$\therefore\; f(z) = (z^2 + 1)(z^2 - 4z + 13)$	M1 A1	
		$f(z) = 0 \Rightarrow z = \pm i$ or $z = \frac{4 \pm \sqrt{16 - 52}}{2}$	M1	
		$\therefore\; z = \pm i$ or $2 \pm 3i$	A1	**(10)**

6.	*(a)*	require $\dfrac{d(r\sin\theta)}{d\theta} = 0$	M1	
		$r\sin\theta = a\sin\theta\sqrt{\sin 2\theta}$	A1	
		$\therefore \dfrac{d(r\sin\theta)}{d\theta} = a[\sin\theta\,\tfrac{1}{2}(\sin 2\theta)^{-\frac{1}{2}}(2\cos 2\theta) + \sqrt{\sin 2\theta}\,(\cos\theta)]$	M1 A1	
		giving $(\sin 2\theta)^{-\frac{1}{2}}[\sin\theta\cos 2\theta + \sin 2\theta\cos\theta] = 0$	M1	
		$(\sin 2\theta)^{-\frac{1}{2}}[\sin 3\theta] = 0$		
		$\therefore \sin 3\theta = 0,\ 3\theta = 0, \pi, 2\pi \ldots$	A1	
		at A, $0 < \theta < \frac{\pi}{2}$ $\therefore \theta = \frac{\pi}{3}$	A1	
	(b)	at A, $r^2 = a^2\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}a^2$	M1	
		side length $= OP = OA\sin\frac{\pi}{3} = \sqrt{\frac{\sqrt{3}}{2}}\,a \times \frac{\sqrt{3}}{2}$	M1	
		area $= OP^2 = \frac{\sqrt{3}}{2}a^2 \times \frac{3}{4} = \frac{3\sqrt{3}}{8}a^2$	A1	
	(c)	area used $= \frac{1}{2}\int_0^{\frac{\pi}{2}} a^2\sin 2\theta\ d\theta$	M1	
		$= \frac{1}{2}a^2[-\frac{1}{2}\cos 2\theta]_0^{\frac{\pi}{2}}$	A1	
		$= -\frac{1}{4}a^2[\cos\pi - \cos 0] = \frac{1}{2}a^2$	A1	
		$\therefore$ area not used $= \frac{3\sqrt{3}}{8}a^2 - \frac{1}{2}a^2 = \frac{1}{8}a^2(3\sqrt{3} - 4)$	M1 A1	**(15)**
7.	*(a)*	$x = ke^{-t},\ \dfrac{dx}{dt} = -ke^{-t},\ \dfrac{d^2x}{dt^2} = ke^{-t}$	M1	
		$\therefore\ ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 8e^{-t}$	M1	
		$2k = 8\ \therefore k = 4$	A1	
	(b)	aux. eqn. $m^2 + 5m + 6 = 0$	M1	
		$(m+3)(m+2) = 0;\ m = -2, -3$	A1	
		gen. soln. $x = Ae^{-2t} + Be^{-3t} + 4e^{-t}$	A1	
		$\dfrac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - 4e^{-t}$	M1	
		$t = 0, x = 1\ \therefore\ 1 = A + B + 4$		
		$t = 0, \dfrac{dx}{dt} = 3\ \therefore\ 3 = -2A - 3B - 4$	A1	
		solve simul. giving $A = -2, B = -1$	M1 A1	
		$\therefore\ x = 4e^{-t} - 2e^{-2t} - e^{-3t}$	A1	
	(c)	$\dfrac{dx}{dt} = -4e^{-t} + 4e^{-2t} + 3e^{-3t}$	M1	
		for S.P. $= 0\ \therefore e^{-t}(3e^{-2t} + 4e^{-t} - 4) = 0$	M1	
		$e^{-t}(3e^{-t} - 2)(e^{-t} + 2) = 0$	M1	
		$\therefore\ e^{-t} = 0$ (no solns) or -2 (no solns) or $\frac{2}{3}$	A1	
		when $e^{-t} = \frac{2}{3},\ x = 4(\frac{2}{3}) - 2(\frac{2}{3})^2 - (\frac{2}{3})^3$	M1	
		$= \frac{8}{3} - \frac{8}{9} - \frac{8}{27} = \frac{40}{27}$	A1	
		max occurs at $T = -\ln\frac{2}{3} = \ln\frac{3}{2}$	A1	**(18)**
			Total	**(75)**

Performance Record – FP1 Paper H

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	series	inequality	1st order diff. eqn	numerical methods	complex nos.	polar coords	2nd order diff. eqn	
Marks	6	8	9	9	10	15	18	75
Student								