GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP3

Paper G

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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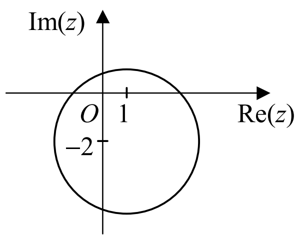
FP3 Paper G – Marking Guide

1.	det $\mathbf{A} = 3(0 + k) - 1(0 + 2) - 4(k - 4)$	M1	
	$= 3k - 2 - 4k + 16 = 14 - k$	A1	
	$\therefore$ singular if $k = 14$	A1	**(3)**

2.	$\lvert -4 + 4\sqrt{3}\,\mathrm{i} \rvert = 4\sqrt{(1 + 3)} = 8$; arg $(-4 + 4\sqrt{3}\,\mathrm{i}) = \arctan(-\sqrt{3}) = \frac{2\pi}{3}$	M1	
	$\therefore (r\mathrm{e}^{\mathrm{i}\theta})^3 = 8\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$	A1	
	$r^3 = 8$ so $r = 2$	A1	
	$3\theta = 2n\pi + \frac{2\pi}{3}$	M1	
	$n = 0, 1, 2$ gives $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}$	A1	
	$\therefore z = 2(\cos\frac{2\pi}{9} + \mathrm{i}\sin\frac{2\pi}{9}), 2(\cos\frac{8\pi}{9} + \mathrm{i}\sin\frac{8\pi}{9}), 2(\cos\frac{14\pi}{9} + \mathrm{i}\sin\frac{14\pi}{9})$	A1	**(6)**

3.	assume true for $n = k$ $\therefore$ $\mathrm{f}(k) = k(k^2 + 5)$ is divisible by 6		
	$\mathrm{f}(k + 1) = (k + 1)[(k + 1)^2 + 5]$	M1	
	$= (k + 1)(k^2 + 2k + 6)$		
	$\mathrm{f}(k + 1) - \mathrm{f}(k) = (k + 1)(k^2 + 2k + 6) - k(k^2 + 5)$	M1	
	$= k^3 + 2k^2 + 6k + k^2 + 2k + 6 - k^3 - 5k$		
	$= 3k^2 + 3k + 6 = 3k(k + 1) + 6$		
	$\therefore \mathrm{f}(k + 1) = 3k(k + 1) + 6 + \mathrm{f}(k)$	A1	
	k, $(k + 1)$ are consec. integers $\therefore$ $k(k + 1)$ is div. by 2 [one must be even]	M1	
	$\therefore$ $3k(k + 1)$ is div. by 6 $\therefore$ $\mathrm{f}(k + 1)$ is div. by 6	A1	
	$\therefore$ true for $n = k + 1$ if true for $n = k$		
	if $n = 1$ $\mathrm{f}(1) = 1 \times 6 = 6$ $\therefore$ $\mathrm{f}(1)$ is div. by 6 $\therefore$ true for $n = 1$	B1	
	$\therefore$ by induction true for $n \in \mathbb{Z}^+$	A1	**(7)**

4.	*(a)* $\lvert z - (1 - 2\mathrm{i}) \rvert = 3$ $\therefore$ circle, centre $1 - 2\mathrm{i}$, radius 3	M1 A1	
		B1	
	(b) T : enlargement s.f 4, centre O		
	giving circle, centre $4 - 8\mathrm{i}$, radius 12	M1 A1	
	U : translation through $5 - \mathrm{i}$		
	giving circle centre $6 - 3\mathrm{i}$, radius 3	M1 A1	
	V : anticlockwise rotation through $\frac{\pi}{2}$ about O		
	giving circle centre $2 + \mathrm{i}$, radius 3	M1 A1	**(9)**

5.	*(a)*	$f(x) = \cos x,\ f'(x) = -\sin x,\ f''(x) = -\cos x,\ f'''(x) = \sin x$	M1 A1	
		$f(\tfrac{\pi}{6}) = \tfrac{\sqrt{3}}{2},\ f'(\tfrac{\pi}{6}) = -\tfrac{1}{2},\ f''(\tfrac{\pi}{6}) = -\tfrac{\sqrt{3}}{2},\ f'''(\tfrac{\pi}{6}) = \tfrac{1}{2}$	A1	
		$f(x) = \tfrac{\sqrt{3}}{2} - \tfrac{1}{2}(x - \tfrac{\pi}{6}) + \tfrac{1}{2!}(-\tfrac{\sqrt{3}}{2})(x - \tfrac{\pi}{6})^2 + \tfrac{1}{2}(\tfrac{1}{3!})(x - \tfrac{\pi}{6})^3 + \ldots$	M1	
		$f(x) = \tfrac{\sqrt{3}}{2} - \tfrac{1}{2}(x - \tfrac{\pi}{6}) - \tfrac{\sqrt{3}}{4}(x - \tfrac{\pi}{6})^2 + \tfrac{1}{12}(x - \tfrac{\pi}{6})^3 + \ldots$	A1	
	(b)	if $x = \tfrac{\pi}{4},\ x - \tfrac{\pi}{6} = \tfrac{\pi}{12}$	M1	
		$\therefore \cos\tfrac{\pi}{4} = \tfrac{\sqrt{3}}{2} - \tfrac{1}{2}(\tfrac{\pi}{12}) - \tfrac{\sqrt{3}}{4}(\tfrac{\pi}{12})^2 + \tfrac{1}{12}(\tfrac{\pi}{12})^3 + \ldots$	M1	
		$= 0.7069$ (4dp)	A1	
	(c)	% error $= \dfrac{\cos\frac{\pi}{4} - 0.7069}{\cos\frac{\pi}{4}} \times 100\% = 0.023\ \%$ (2sf)	M1 A1	**(10)**

6.	*(a)*	$\dfrac{d^3y}{dx^3} = 2x + x\dfrac{dy}{dx} + y - 2y\dfrac{dy}{dx}$	M1 A1	
		$x_0 = 0,\ y_0 = \tfrac{1}{2},\ \left(\dfrac{dy}{dx}\right)_0 = -1;\ \left(\dfrac{d^2y}{dx^2}\right)_0 = 0 + 0 - \tfrac{1}{4} = -\tfrac{1}{4}$	A1	
		$\left(\dfrac{d^3y}{dx^3}\right)_0 = 0 + 0 + \tfrac{1}{2} - [2 \times \tfrac{1}{2} \times (-1)] = \tfrac{3}{2}$	A1	
		$\therefore\ y = \tfrac{1}{2} - 1x + \tfrac{1}{2!}(-\tfrac{1}{4})x^2 + \tfrac{1}{3!}(\tfrac{3}{2})x^3 + \ldots$	M1	
		$y = \tfrac{1}{2} - x - \tfrac{1}{8}x^2 + \tfrac{1}{4}x^3 + \ldots$	A1	
	(b)	$x = -0.1,\ y \approx 0.5 + 0.1 - 0.00125 - 0.00025 = 0.5985$	A1	
	(c)	$\dfrac{y_1 - 2y_0 + y_{-1}}{0.01} = x_0^2 + x_0y_0 - y_0^2$	M1	
		$y_1 - 2y_0 + y_{-1} = 0.01(x_0^2 + x_0y_0 - y_0^2)$		
		$y_1 = 2y_0 - y_{-1} + 0.01(x_0^2 + x_0y_0 - y_0^2)$	A1	
		$x_{-1} = -0.1,\ x_0 = 0,\ x_1 = 0.1;\ y_{-1} = 0.5985,\ y_0 = 0.5,\ y_1 = ?$		
		$y_1 = 1 - 0.5985 + 0.01(0 + 0 - 0.25) = 0.399$	A1	**(10)**

7. *(a)* $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -4 & 1 \\ 4 & -2 & 5 \end{vmatrix}$

$= \mathbf{i}(-20 + 2) - \mathbf{j}(10 - 4) + \mathbf{k}(-4 + 16) = -18\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$ M1 A1

$|\, \mathbf{a} \times \mathbf{b} \,| = |\, \mathbf{a} \,|\,|\, \mathbf{b} \,| \sin \theta$

$6\sqrt{(9 + 1 + 4)} = \sqrt{21}\sqrt{45} \sin \theta$ M1 A1

$6\sqrt{7}\sqrt{2} = 3\sqrt{7}\sqrt{3}\sqrt{5}\sin \theta$

$\sin \theta = \frac{2\sqrt{2}}{\sqrt{15}}$ or $\frac{2}{15}\sqrt{30}$ A1

(b) $\mathbf{n} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ M1

$\mathbf{r}.(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}).(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 6 - 1 - 4 = 1$ M1 A1

$\mathbf{r}.(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 1 \;\; \therefore\; 3x + y - 2z - 1 = 0$ A1

(c) $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + u(2\mathbf{j} + \mathbf{k})$

$u = 0,\ \mathbf{r} = \mathbf{i} - 2\mathbf{j},\ (\mathbf{i} - 2\mathbf{j}).(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 3 - 2 = 1 \;\therefore$ in plane M1 A1

$u = 1,\ \mathbf{r} = \mathbf{i} + \mathbf{k},\ (\mathbf{i} + \mathbf{k}).(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 3 - 2 = 1 \;\therefore$ in plane M1

two points on line in plane $\therefore$ line in plane A1 **(13)**

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8. *(a)* $(\mathbf{AB})(\mathbf{B}^{-1}\mathbf{A}^{-1}) = \mathbf{ABB}^{-1}\mathbf{A}^{-1} = \mathbf{AIA}^{-1} = \mathbf{AA}^{-1} = \mathbf{I}$ M1 A1

$\therefore (\mathbf{B}^{-1}\mathbf{A}^{-1})$ is inverse of $(\mathbf{AB})$ i.e. $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ M1 A1

(b) $S(a_1\mathbf{v}_1 + a_2\mathbf{v}_2) = S\begin{pmatrix} a_1x_1 + a_2x_2 \\ a_1y_1 + a_2y_2 \end{pmatrix}$ M1

$= \begin{pmatrix} a_1y_1 + a_2y_2 - a_1x_1 - a_2x_2 \\ 2a_1x_1 + 2a_2x_2 + a_1y_1 + a_2y_2 \end{pmatrix}$ M1 A1

$= \begin{pmatrix} a_1(y_1 - x_1) + a_2(y_2 - x_2) \\ a_1(2x_1 + y_1) + a_2(2x_2 + y_2) \end{pmatrix}$ M1

$= a_1\begin{pmatrix} y_1 - x_1 \\ 2x_1 + y_1 \end{pmatrix} + a_2\begin{pmatrix} y_2 - x_2 \\ 2x_2 + y_2 \end{pmatrix}$ A1

$= a_1S(\mathbf{v}_1) + a_2S(\mathbf{v}_2) \;\therefore\; S$ is a linear transformation M1 A1

(c) $\mathbf{S} = \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix},\ \mathbf{T} = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$ M1 A1

$\mathbf{ST} = \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 7 & 1 \end{pmatrix}$ M1 A1

$\det(\mathbf{ST}) = -2 - 7 = -9$ M1

$\therefore (\mathbf{ST})^{-1} = -\frac{1}{9}\begin{pmatrix} 1 & -1 \\ -7 & -2 \end{pmatrix}$ or $\frac{1}{9}\begin{pmatrix} -1 & 1 \\ 7 & 2 \end{pmatrix}$ A1 **(17)**

Total **(75)**

Performance Record – FP3 Paper G

Question no.	1	2	3	4	5	6	7	8	Total
Topic(s)	singular matrix	complex nos.	proof by induction	complex trans.	Taylor series	diff. eqn., Taylor series, step-by-step	vectors	matrices, linear trans.	
Marks	3	6	7	9	10	10	13	17	75
Student									