## GCE Examinations

## Advanced / Advanced Subsidiary

## Core Mathematics C2

## Paper E

Time: 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72 .
- You are reminded of the need for clear presentation in your answers.

1. A sequence of terms is defined by

$$
u_{n}=3^{n}-2, \quad n \geq 1 .
$$

(i) Write down the first four terms of the sequence.

The same sequence can also be defined by the recurrence relation

$$
u_{n+1}=a u_{n}+b, \quad n \geq 1, \quad u_{1}=1,
$$

where $a$ and $b$ are constants.
(ii) Find the values of $a$ and $b$.
2.


The diagram shows the curve with equation $y=\sqrt{4 x-1}$.
(i) Use the trapezium rule with four intervals of equal width to estimate the area of the shaded region bounded by the curve, the $x$-axis and the lines $x=1$ and $x=3$.
(ii) Explain briefly how you could use the trapezium rule to obtain a more accurate estimate of the area of the shaded region.
3. (i) Expand $(2+y)^{6}$ in ascending powers of $y$ as far as the term in $y^{3}$, simplifying each coefficient.
(ii) Hence expand $\left(2+x-x^{2}\right)^{6}$ in ascending powers of $x$ as far as the term in $x^{3}$, simplifying each coefficient.
4.

$$
\mathrm{f}(x)=\frac{4}{2+\sin x^{\circ}}
$$

(i) State the maximum value of $\mathrm{f}(x)$ and the smallest positive value of $x$ for which $\mathrm{f}(x)$ takes this value.
(ii) Solve the equation $\mathrm{f}(x)=3$ for $0 \leq x \leq 360$, giving your answers to 1 decimal place.
5. (a) Given that $t=\log _{3} x$,
(i) write down an expression in terms of $t$ for $\log _{3} x^{2}$,
(ii) show that $\log _{9} x=\frac{1}{2} t$.
(b) Hence, or otherwise, find to 3 significant figures the value of $x$ such that

$$
\begin{equation*}
\log _{3} x^{2}-\log _{9} x=4 \tag{3}
\end{equation*}
$$

6. Given that

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{x^{3}-4}{x^{3}}, x \neq 0 \tag{8}
\end{equation*}
$$

and that $y=0$ when $x=-1$, find the value of $y$ when $x=2$.
7. A student completes a mathematics course and begins to work through past exam papers. He completes the first paper in 2 hours and the second in 1 hour 54 minutes.

Assuming that the times he takes to complete successive papers form a geometric sequence,
(i) find, to the nearest minute, how long he will take to complete the fifth paper,
(ii) show that the total time he takes to complete the first eight papers is approximately 13 hours 28 minutes,
(iii) find the least number of papers he must work through if he is to complete a paper in less than one hour.
8.


The diagram shows a circle of radius 12 cm which passes through the points $P$ and $Q$. The chord $P Q$ subtends an angle of $120^{\circ}$ at the centre of the circle.
(i) Find the exact length of the major arc $P Q$.
(ii) Show that the perimeter of the shaded minor segment is given by $k(2 \pi+3 \sqrt{3}) \mathrm{cm}$, where $k$ is an integer to be found.
(iii) Find, to 1 decimal place, the area of the shaded minor segment as a percentage of the area of the circle.
9.

$$
\mathrm{f}(x)=x^{3}-9 x^{2}+24 x-16
$$

(i) Evaluate $\mathrm{f}(1)$ and hence state a linear factor of $\mathrm{f}(x)$.
(ii) Show that $\mathrm{f}(x)$ can be expressed in the form

$$
\mathrm{f}(x)=(x+p)(x+q)^{2},
$$

where $p$ and $q$ are integers to be found.
(iii) Sketch the curve $y=\mathrm{f}(x)$.
(iv) Using integration, find the area of the region enclosed by the curve $y=\mathrm{f}(x)$ and the $x$-axis.

