FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

# Core Mathematics C3

Paper A

Time: 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

## INFORMATION FOR CANDIDATES

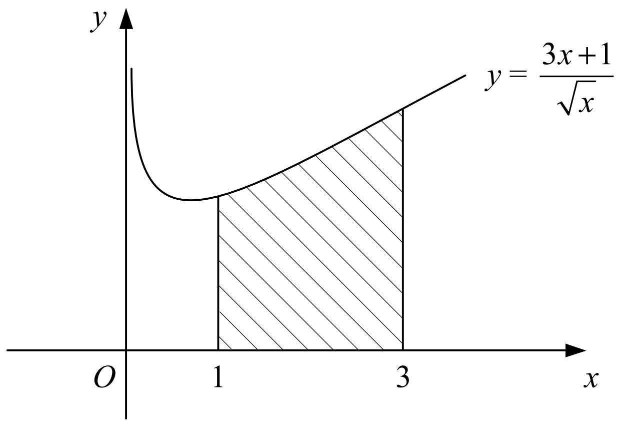
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

*Written by Shaun Armstrong*

**1.** Evaluate

$$\int_2^{15} \frac{1}{\sqrt[3]{2x-3}}\ \mathrm{d}x. \qquad [5]$$

**2.**




The diagram shows the curve with equation $y = \dfrac{3x+1}{\sqrt{x}}$, $x > 0$.

The shaded region is bounded by the curve, the $x$-axis and the lines $x = 1$ and $x = 3$.

Find the volume of the solid formed when the shaded region is rotated through four right angles about the $x$-axis, giving your answer in the form $\pi(a + \ln b)$, where $a$ and $b$ are integers. [6]

**3.** A curve has the equation $y = (3x - 5)^3$.

*(i)* Find an equation for the tangent to the curve at the point $P$ (2, 1). [4]

The tangent to the curve at the point $Q$ is parallel to the tangent at $P$.

*(ii)* Find the coordinates of $Q$. [3]

**4.** Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} - x + 2 = 0$$

$$\ln (x + 3) - 2y - 1 = 0 \qquad [7]$$

5. *(i)* Find the exact value of $x$ such that

$$3 \tan^{-1} (x - 2) + \pi = 0.$$ [3]

*(ii)* Solve, for $-\pi < \theta < \pi$, the equation

$$\cos 2\theta - \sin \theta - 1 = 0,$$

giving your answers in terms of $\pi$. [5]

6. The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 3x - 4, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \rightarrow \frac{2}{x+3}, \quad x \in \mathbb{R}, \quad x \neq -3.$$

*(i)* Evaluate fg(1). [2]

*(ii)* Solve the equation $\mathrm{gf}(x) = 6$. [4]

*(iii)* Find an expression for $\mathrm{g}^{-1}(x)$. [2]

7. *(i)* Express $2 \sin x° - 3 \cos x°$ in the form $R \sin (x - \alpha)°$ where $R > 0$ and $0 < \alpha < 90$. [3]

*(ii)* Show that the equation

$$\operatorname{cosec} x° + 3 \cot x° = 2$$

can be written in the form

$$2 \sin x° - 3 \cos x° = 1.$$ [1]

*(iii)* Solve the equation

$$\operatorname{cosec} x° + 3 \cot x° = 2,$$

for $x$ in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. [4]

***Turn over***

**8.** The functions f and g are defined for all real values of $x$ by

$$\text{f} : x \to |x - 3a|,$$

$$\text{g} : x \to |2x + a|,$$

where $a$ is a positive constant.

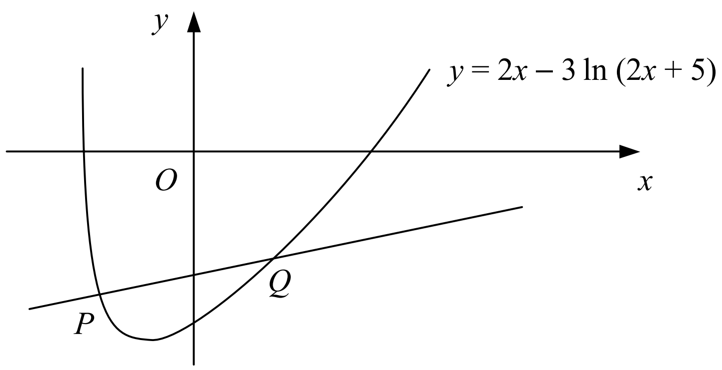
*(i)* Evaluate $\text{fg}(-2a)$. [2]

*(ii)* Sketch on the same diagram the graphs of $y = \text{f}(x)$ and $y = \text{g}(x)$, showing the coordinates of any points where each graph meets the coordinate axes. [4]

*(iii)* Solve the equation

$$\text{f}(x) = \text{g}(x).$$ [4]

**9.**




The diagram shows the curve with equation $y = 2x - 3 \ln (2x + 5)$ and the normal to the curve at the point $P$ (–2, –4).

*(i)* Find an equation for the normal to the curve at $P$. [4]

The normal to the curve at $P$ intersects the curve again at the point $Q$ with $x$-coordinate $q$.

*(ii)* Show that $1 < q < 2$. [3]

*(iii)* Show that $q$ is a solution of the equation

$$x = \tfrac{12}{7} \ln (2x + 5) - 2.$$ [2]

*(iv)* Use an iterative process based on the equation above with a starting value of 1.5 to find the value of $q$ to 3 significant figures and justify the accuracy of your answer. [4]