FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C3

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper E – Marking Guide

1. *(i)* $0.17 < x < 0.23$ M1 A1

(ii) $0.17 < 0.95^n < 0.23$, $\ln 0.17 < n \ln 0.95 < \ln 0.23$ M1

$\frac{\ln 0.17}{\ln 0.95} > n > \frac{\ln 0.23}{\ln 0.95}$, $28.65 < n < 34.55$ M1

$\therefore\ n = 29, 30, 31, 32, 33, 34$ A1 **(5)**

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2. $= \pi\int_0^2 x^2(2-x)\ dx = \pi\int_0^2 (2x^2 - x^3)\ dx$ M1

$= \pi[\frac{2}{3}x^3 - \frac{1}{4}x^4]_0^2$ M1 A1

$= \pi[(\frac{16}{3} - 4) - (0)] = \frac{4}{3}\pi$ M1 A1 **(5)**

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3. $2(\text{cosec}^2 y - 1) + 5\ \text{cosec}\ y + \text{cosec}^2 y = 0$ M1

$3\ \text{cosec}^2 y + 5\ \text{cosec}\ y - 2 = 0$

$(3\ \text{cosec}\ y - 1)(\text{cosec}\ y + 2) = 0$ M1

$\text{cosec}\ y = -2$ or $\frac{1}{3}$ (no solutions) A1

$\sin y = -\frac{1}{2}$ M1

$y = 180 + 30, 360 - 30$

$y = 210, 330$ A2 **(6)**

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4. *(i)* $\frac{dx}{dy} = 1 \times \sqrt{1-2y} + y \times \frac{1}{2}(1-2y)^{-\frac{1}{2}} \times (-2)$ M1 A1

$= \sqrt{1-2y} - \frac{y}{\sqrt{1-2y}} = \frac{(1-2y)-y}{\sqrt{1-2y}} = \frac{1-3y}{\sqrt{1-2y}}$ M1

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\sqrt{1-2y}}{1-3y}$ A1

(ii) $y = -1,\ x = -\sqrt{3}$, grad $= \frac{1}{4}\sqrt{3}$ B1

$\therefore\ y + 1 = \frac{1}{4}\sqrt{3}\,(x + \sqrt{3}\,)$ M1

$4y + 4 = \sqrt{3}\,x + 3$

$\sqrt{3}\,x - 4y - 1 = 0$ $[p = -4, q = -1]$ A1 **(7)**

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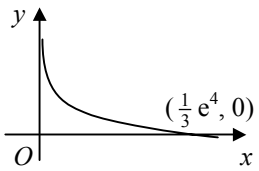
5. *(i)* $4 - \ln 3x = 0$

$\ln 3x = 4$

$x = \frac{1}{3}e^4$ M1 A1

(ii)



B2

(iii) $fg(x) = 4 - \ln 3e^{2-x} = 4 - (\ln 3 + \ln e^{2-x})$ M1

$= 4 - \ln 3 - (2 - x)$ M1

$= x + 2 - \ln 3$ $[a = 2, b = 3]$ A1 **(7)**

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6. *(i)* $= [-\frac{1}{2}e^{1-2x}]_{-1}^{0}$ M1 A1

$= -\frac{1}{2}(e - e^3) = \frac{1}{2}e(e^2 - 1)$ M1 A1

(ii) $= \int_2^4 (3x - \frac{2}{x})\ dx = [\frac{3}{2}x^2 - 2\ln|x|]_2^4$ M1 A1

$= (24 - 2\ln 4) - (6 - 2\ln 2) = 18 - 2\ln 2$ M1 A1 **(8)**

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7. *(i)* $a = 2,\ \cos x + 3\sin x = b\cos x\cos c + b\sin x\sin c$

$b\cos c = 1,\ b\sin c = 3$ M1

$\therefore\ b = \sqrt{1^2 + 3^2} = \sqrt{10}$ A1

$\tan c = 3,\ c = 1.25$ (3sf) A1

$\therefore\ \mathrm{f}(x) = 2 + \sqrt{10}\cos(x - 1.25)$

(ii) $2 + \sqrt{10}\cos(x - 1.249) = 0, \quad \cos(x - 1.249) = -\frac{2}{\sqrt{10}}$ M1

$x - 1.249 = \pi - 0.8861,\ \pi + 0.8861 = 2.256,\ 4.028$ M1

$x = 3.50,\ 5.28$ (3sf) A2

(iii)

x	0	0.5	1	1.5	2
$\mathrm{f}(x)$	3	4.3159	5.0647	5.0632	4.3117

M1

$I \approx \frac{1}{3} \times 0.5 \times [3 + 4.3117 + 4(4.3159 + 5.0632) + 2(5.0647)]$ M1

$= 9.16$ (3sf) A1 **(10)**

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8. *(i)* $= 2[x^2 + 2x] + 2 = 2[(x+1)^2 - 1] + 2$ M1

$= 2(x+1)^2$ A1

(ii) translation by 1 unit in negative x direction

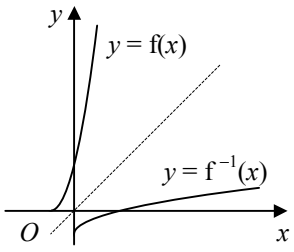
stretch by scale factor of 2 in y direction (either first) B3

(iii) $y = 2(x+1)^2, \quad \frac{y}{2} = (x+1)^2$

$x + 1 = \pm\sqrt{\frac{y}{2}}, \quad x = -1 \pm \sqrt{\frac{y}{2}}$ M1

$\therefore\ \mathrm{f}^{-1}(x) = -1 + \sqrt{\frac{x}{2}},\ x \in \mathbb{R},\ x \geq 0$ A2

(iv)



B2

$y = \mathrm{f}^{-1}(x)$ is reflection of

$y = \mathrm{f}(x)$ in line $y = x$ B1 **(11)**

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9. *(i)* $t = 10,\ T = 18 \quad \Rightarrow \quad 18 = 5 + Ae^{-10k}$ (1)

$t = 60,\ T = 12 \quad \Rightarrow \quad 12 = 5 + Ae^{-60k}$ (2) M1

(1) $\Rightarrow \quad A = \frac{13}{e^{-10k}} = 13e^{10k}$ M1

sub (2) $\Rightarrow \quad 7 = 13e^{10k} \times e^{-60k}$

$e^{-50k} = \frac{7}{13}$ A1

$\therefore\ k = -\frac{1}{50}\ln\frac{7}{13} = 0.0124$ (3sf) M1 A1

$\therefore\ A = 13e^{10 \times 0.01238} = 14.7$ (3sf) A1

(ii) $T = 5 + 14.71e^{-0.01238t}$

$\frac{dT}{dt} = -0.01238 \times 14.71\ e^{-0.01238t} = -0.1822e^{-0.01238t}$ M1 A1

when $t = 20, \quad \frac{dT}{dt} = -0.1822e^{-0.01238 \times 20} = -0.142$ M1

$\therefore$ temperature decreasing at rate of 0.142 °C per minute (3sf) A1

(iii) $T = 5 + 14.71e^{-0.01238(t-60)}$ M1

$= 5 + 14.71e^{0.7428 - 0.01238t}$

$= 5 + 14.71e^{0.7428} \times e^{-0.01238t}$ M1

$= 5 + 30.9e^{-0.01238t}, \quad B = 30.9$ (3sf) A1 **(13)**

Total **(72)**

Performance Record – C3 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	functions	integration	trigonometry	differentiation	functions	integration	trigonometry, Simpson's rule	functions	exponentials and logarithms	
Marks	5	5	6	7	7	8	10	11	13	72
Student										