FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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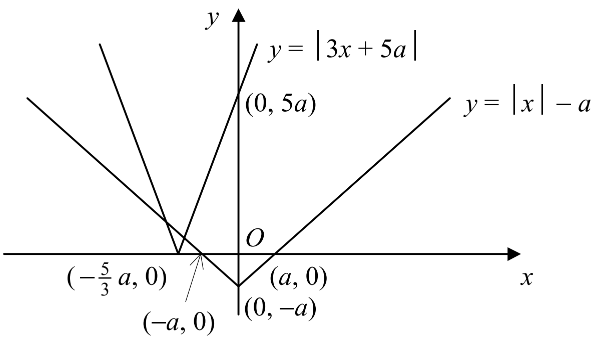
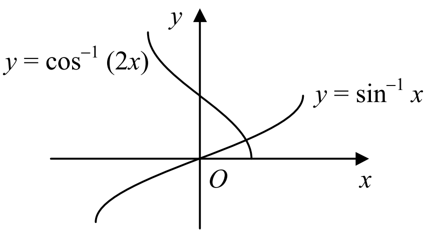
C3 Paper F – Marking Guide

1.		$= [\frac{2}{9}(3x-2)^{\frac{3}{2}}]_2^6$	M1 A1	
		$= \frac{2}{9}(64 - 8) = 12\frac{4}{9}$	M1 A1	**(4)**
2.	*(i)*	$= -3(2x-7)^{-\frac{3}{2}} \times 2 = -\frac{6}{(2x-7)^{\frac{3}{2}}}$	M1 A1	
	(ii)	$= 2x \times e^{-x} + x^2 \times (-e^{-x}) = xe^{-x}(2 - x)$	M1 A2	**(5)**
3.	*(i)*	LHS $\equiv \sqrt{2}(\cos x \cos 45 - \sin x \sin 45) + 2(\cos x \cos 30 + \sin x \sin 30)$	M1 A1	
		$\equiv \sqrt{2}(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x) + 2(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x)$	M1	
		$\equiv \cos x - \sin x + \sqrt{3}\cos x + \sin x \equiv (1 + \sqrt{3})\cos x \equiv$ RHS	A1	
	(ii)	let $x = 75$, $\sqrt{2}\cos 120° + 2\cos 45° = (1 + \sqrt{3})\cos 75°$	M1	
		$\sqrt{2}(-\frac{1}{2}) + 2(\frac{1}{\sqrt{2}}) = (1 + \sqrt{3})\cos 75°$		
		$\frac{1}{2}\sqrt{2} = (1 + \sqrt{3})\cos 75°$	M1	
		$\cos 75° = \frac{\sqrt{2}}{2(1+\sqrt{3})} = \frac{1}{\sqrt{2}+\sqrt{6}}$	A1	**(7)**
4.	*(i)*	$f(1) = 2.30$, $f(1.5) = -18.5$	M1	
		sign change, f(x) continuous $\therefore$ root	A1	
	(ii)	$x^2 + 5x - 2\sec x = 0 \Rightarrow x^2 + 5x = \frac{2}{\cos x}$	M1	
		$\cos x = \frac{2}{x^2+5x}$	M1	
		$x = \cos^{-1}(\frac{2}{x^2+5x}) \therefore x_{n+1} = \cos^{-1}(\frac{2}{x_n^2+5x_n})$	A1	
	(iii)	$x_0 = 1.25$, $x_1 = 1.31191$, $x_2 = 1.32686$, $x_3 = 1.33024$,	M1 A1	
		$x_4 = 1.33100$, $x_5 = 1.33116$, $x_6 = 1.33120$ $\therefore$ $\alpha = 1.331$ (3dp)	A1	**(8)**
5.	*(i)*	$= f(2) = 2 + \ln 4$	M1 A1	
	(ii)	$f'(x) = \frac{1}{3x-2} \times 3 = \frac{3}{3x-2}$	M1	
		$x = 1$, $y = 2$, grad $= 3$	A1	
		$y - 2 = 3(x - 1)$ $[y = 3x - 1]$	M1 A1	
	(iii)	$y = 2 + \ln(3x - 2)$, $3x - 2 = e^{y-2}$	M1	
		$x = \frac{1}{3}(2 + e^{y-2})$, $f^{-1}(x) = \frac{1}{3}(2 + e^{x-2})$	A1	**(8)**
6.	*(i)*		B3 B2	
	(ii)	$-x - a = 3x + 5a \Rightarrow x = -\frac{3}{2}a$	M1 A1	
		$-x - a = -(3x + 5a) \Rightarrow x = -2a$, $x = -2a, -\frac{3}{2}a$	M1 A1	**(9)**

7.	*(i)*	$= \int_2^4 (2x - e^{\frac{1}{2}x})\ dx$		
		$= [x^2 - 2e^{\frac{1}{2}x}]_2^4$	M1 A1	
		$= (16 - 2e^2) - (4 - 2e) = 12 + 2e - 2e^2$	M1 A1	
	(ii)	$V = \pi \int_2^4 (2x - e^{\frac{1}{2}x})^2\ dx$	M1	
		x 2 3 4		
		$(2x - e^{\frac{1}{2}x})^2$ 1.6428 2.3053 0.3733	M1	
		$I \approx \frac{1}{3} \times 1 \times [1.6428 + 0.3733 + 2(2.3053)] = 3.7458$	M1 A1	
		$\therefore\ V \approx 3.7458\pi = 11.8$ (3sf)	A1	**(9)**

8.	*(i)*		B3	
	(ii)	$b = \sin^{-1} a \quad \Rightarrow \quad a = \sin b$		
		$b = \cos^{-1} 2a \quad \Rightarrow \quad 2a = \cos b$	M1	
		$\therefore\ 2 \sin b = \cos b$	M1	
		$\frac{\sin b}{\cos b} = \frac{1}{2}$		
		$\tan b = \frac{1}{2}$	A1	
	(iii)	$\tan^2 b = \frac{1}{4}$		
		$\sec^2 b = 1 + \frac{1}{4} = \frac{5}{4}$	M1	
		$\cos^2 b = \frac{4}{5}$		
		$\cos b = \pm\frac{2}{\sqrt{5}}$	A1	
		$a = \frac{1}{2}\cos b = \pm\frac{1}{\sqrt{5}}$	M1	
		from diagram, $a > 0\ \therefore\ a = \frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$	A1	**(10)**

9.	*(i)*	$f(x) > -2$	B1	
	(ii)	$x = 0,\ y = e - 2\ \therefore\ P\,(0, e - 2)$	B1	
		$y = 0,\ 0 = e^{3x+1} - 2$		
		$3x + 1 = \ln 2$	M1	
		$x = \frac{1}{3}(\ln 2 - 1)\ \therefore\ Q\,(\frac{1}{3}(\ln 2 - 1), 0)$	A1	
	(iii)	$f'(x) = 3e^{3x+1}$	M1	
		at P, grad $= 3e$	A1	
		$\therefore\ y - (e - 2) = 3e(x - 0)$	M1	
		$y = 3ex + e - 2$	A1	
	(iv)	at Q, grad $= 6$	B1	
		tangent at Q: $y - 0 = 6(x - \frac{1}{3}(\ln 2 - 1))$	M1	
		$y = 6x - 2\ln 2 + 2$		
		intersect: $3ex + e - 2 = 6x - 2\ln 2 + 2$		
		$x(3e - 6) = 4 - e - 2\ln 2$	M1	
		$x = \frac{4 - e - 2\ln 2}{3e - 6} = -0.0485$ (3sf)	A1	**(12)**

Total **(72)**

Performance Record – C3 Paper F

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	Total
Topic(s)	integration	differentiation	trigonometry	numerical methods	functions	functions	integration, Simpson's rule	trigonometry	functions, exponentials and logarithms	
Marks	4	5	7	8	8	9	9	10	12	72
Student										