FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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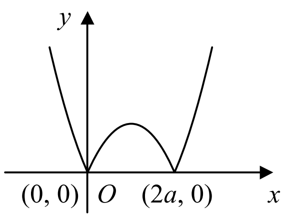
C3 Paper H – Marking Guide

1.		$f'(x) = \frac{4 \times (2x+1) - (4x-1) \times 2}{(2x+1)^2} = \frac{6}{(2x+1)^2}$	M1 A1	
		$x = -2 \Rightarrow y = 3,\ \text{grad} = \frac{2}{3}$	A1	
		$\therefore\ y - 3 = \frac{2}{3}(x + 2)$	M1	
		$3y - 9 = 2x + 4$		
		$2x - 3y + 13 = 0$	A1	**(5)**
2.	*(i)*	$x = \frac{1}{3}e^{2y}$	M1 A1	
	(ii)	$= \pi \int_0^1 \frac{1}{9}e^{4y}\ dy$	M1	
		$= \pi[\frac{1}{36}e^{4y}]_0^1$	M1 A1	
		$= \frac{1}{36}\pi(e^4 - 1)$	M1 A1	**(7)**
3.	*(i)*	LHS $\equiv \sin(2x + x)$		
		$\equiv \sin 2x \cos x + \cos 2x \sin x$	M1	
		$\equiv 2 \sin x \cos^2 x + \sin x(1 - 2\sin^2 x)$	M1	
		$\equiv 2 \sin x (1 - \sin^2 x) + \sin x - 2\sin^3 x$	M1	
		$\equiv 2 \sin x - 2\sin^3 x + \sin x - 2\sin^3 x$		
		$\equiv 3 \sin x - 4\sin^3 x \equiv$ RHS	A1	
	(ii)	$3 \sin x - 4 \sin^3 x - \sin x = 0$		
		$2 \sin x(1 - 2\sin^2 x) = 0$	M1	
		$\sin x = 0, \pm\frac{1}{\sqrt{2}}$	A1	
		$x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$	A2	**(8)**
4.	*(i)*		B3	
	(ii)	$= f(3a^2) = 9a^4 - 6a^3$	M1 A1	
	(iii)	$gf(x) = 3a(x^2 - 2ax)$	M1	
		$\therefore\ 3a(x^2 - 2ax) = 9a^3$		
		$x^2 - 2ax - 3a^2 = 0$	A1	
		$(x + a)(x - 3a) = 0$	M1	
		$x = -a, 3a$	A1	**(9)**
5.	*(i)*	$(e^x - 3)(e^x - 5) = 0$	M1	
		$e^x = 3, 5$		
		$x = \ln 3, \ln 5$	M1 A1	
	(ii)	assume $\log_2 3$ is rational	B1	
		$\therefore\ \log_2 3 = \frac{p}{q}$ where p and q are integers and $q \neq 0$	M1	
		$\Rightarrow\ 2^{\frac{p}{q}} = 3$	M1	
		$\Rightarrow\ 2^p = 3^q$	A1	
		2 and 3 are co-prime $\therefore$ only solution is $p = q = 0$	M1	
		but $q \neq 0$ $\therefore$ contradiction $\therefore$ $\log_2 3$ is irrational	A1	**(9)**

6. *(i)* $2x^2 + 3\ln(2-x) = 0 \;\Rightarrow\; 3\ln(2-x) = -2x^2$

$\ln(2-x) = -\tfrac{2}{3}x^2$ — M1

$2 - x = e^{-\frac{2}{3}x^2}$ — M1

$x = 2 - e^{-\frac{2}{3}x^2}$ $[\,k = -\tfrac{2}{3}\,]$ — A1

(ii) $x_1 = 1.90988,\ x_2 = 1.91212,\ x_3 = 1.91262,\ x_4 = 1.91273,\ x_5 = 1.91275$ — M1 A1

$\therefore\ \alpha = 1.913$ (3dp) — A1

(iii) $f'(x) = 4x + \dfrac{3}{2-x} \times (-1) = 4x - \dfrac{3}{2-x}$ — M1 A1

$\therefore\ 4x - \dfrac{3}{2-x} = 0,\quad 4x = \dfrac{3}{2-x},\quad 4x(2-x) = 3$ — M1

$4x^2 - 8x + 3 = 0,\quad (2x-3)(2x-1) = 0$ — M1

$x = \tfrac{1}{2},\ \tfrac{3}{2}$ — A1 **(11)**

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7. *(i)* $\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$

let $A = B = \frac{x}{2}$ $\quad \cos x \equiv \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$ — M1

$\cos x \equiv (1 - \sin^2\frac{x}{2}) - \sin^2\frac{x}{2}$

$\cos x \equiv 1 - 2\sin^2\frac{x}{2}$ — A1

(ii) $\text{LHS} \equiv \dfrac{1-(1-2\sin^2\frac{x}{2})}{2\sin\frac{x}{2}\cos\frac{x}{2}}$ — M1

$\equiv \dfrac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} \equiv \dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$ — M1

$\equiv \tan\frac{x}{2} \equiv \text{RHS}$ — A1

(iii) $\tan\frac{x}{2} = 2\sec^2\frac{x}{2} - 5,\quad \tan\frac{x}{2} = 2(1+\tan^2\frac{x}{2}) - 5$ — M1

$2\tan^2\frac{x}{2} - \tan\frac{x}{2} - 3 = 0,\quad (2\tan\frac{x}{2} - 3)(\tan\frac{x}{2} + 1) = 0$ — M1

$\tan\frac{x}{2} = -1$ or $\frac{3}{2}$ — A1

$\frac{x}{2} = 135$ or 56.310 — M1

$x = 112.6°$ (1dp), $270°$ — A2 **(11)**

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8. *(i)* $f(x) = (x-1)^2 - 1 + 5 = (x-1)^2 + 4$ — M1 A1

(ii) $f(x) \geq 4$ — B1

(iii) $y = (x-1)^2 + 4$

$(x-1)^2 = y - 4$

$x - 1 = \pm\sqrt{y-4}$ — M1

$x = 1 \pm \sqrt{y-4}$

$f^{-1}(x) = 1 + \sqrt{x-4}$ — A1

(iv) translation by 4 units in negative x direction
translation by 1 unit in negative y direction (either first) — B3

(v) $\dfrac{dy}{dx} = \tfrac{1}{2}(x-4)^{-\frac{1}{2}}$ — M1

$x = 8,\ y = 3,\ \text{grad} = \tfrac{1}{4}$ — A1

$\therefore$ grad of normal $= -4$

$\therefore\ y - 3 = -4(x-8)$ $[\,y = 35 - 4x\,]$ — M1 A1 **(12)**

Total **(72)**

Performance Record – C3 Paper H

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	differentiation	integration	trigonometry	functions	exponentials and logarithms	numerical methods, differentiation	trigonometry	functions	
Marks	5	7	8	9	9	11	11	12	72
Student									