FOR OCR

GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C3

Paper L

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for using a valid method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper L – Marking Guide

1.	*(i)*	$= 3x^2 \times \ln x + x^3 \times \frac{1}{x} = x^2(3\ln x + 1)$	M1 A1	
	(ii)	$\frac{dx}{dy} = \frac{1\times(3-2y)-(y+1)\times(-2)}{(3-2y)^2} = \frac{5}{(3-2y)^2}$	M1 A1	
		$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{5}(3-2y)^2$	M1 A1	**(6)**
2.	*(i)*	$A\ (0, 5),\ B\ (0, e^2)$	B2	
	(ii)	$3 + 2e^x = e^{x+2} = e^2e^x$	M1	
		$3 = e^x(e^2 - 2), \quad e^x = \frac{3}{e^2-2}$	M1	
		$x = \ln \frac{3}{e^2-2}$	A1	
		$\therefore\ y = e^2e^x = e^2 \times \frac{3}{e^2-2} = \frac{3e^2}{e^2-2}$	M1 A1	**(7)**
3.	*(i)*	$= g(5) = \log_2 16 = 4$	M1 A1	
	(ii)	$y = \log_2(3x+1), \quad 3x + 1 = 2^y$	M1	
		$x = \frac{1}{3}(2^y - 1), \quad g^{-1}(x) = \frac{1}{3}(2^x - 1)$	A1	
	(iii)	$fg^{-1}(x) = f\left[\frac{1}{3}(2^x - 1)\right] = 2(2^x - 1) - 1 = 2(2^x) - 3$	M1	
		$\therefore\ 2(2^x) - 3 = 2, \quad 2^x = \frac{5}{2}$	A1	
		$x = \frac{\ln\frac{5}{2}}{\ln 2}$ or $\frac{\ln 5 - \ln 2}{\ln 2}$	M1 A1	**(8)**
4.	*(i)*	$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$		
		let $A = B = x \quad \cos 2x \equiv \cos^2 x - \sin^2 x$	M1	
		$\cos 2x \equiv \cos^2 x - (1 - \cos^2 x)$		
		$\cos 2x \equiv 2\cos^2 x - 1$	A1	
	(ii)	$\text{LHS} \equiv 2\cos x - \frac{1}{\cos x} \equiv \frac{2\cos^2 x - 1}{\cos x}$	M1	
		$\equiv \frac{\cos 2x}{\cos x} \equiv \sec x \cos 2x \equiv \text{RHS}$	M1 A1	
	(iii)	$\sec x\cos 2x = 2\cos 2x$		
		$\cos 2x(\sec x - 2) = 0$	M1	
		$\cos 2x = 0$ or $\sec x = 2$	A1	
		$2x = 90, 270$ or $\cos x = \frac{1}{2}$		
		$x = 45°, 60°, 135°$	A2	**(9)**
5.	*(i)*	$2\sin x = -\frac{1}{\cos(x+\frac{\pi}{6})}, \quad 2\sin x\cos(x + \frac{\pi}{6}) = -1$	M1	
		$2\sin x\left[\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6}\right] = -1$	M1	
		$2\sin x\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right] = -1$		
		$\sqrt{3}\sin x\cos x - \sin^2 x = -1$	A1	
		$\sqrt{3}\sin x\cos x - (1 - \cos^2 x) = -1$	M1	
		$\sqrt{3}\sin x\cos x + \cos^2 x = 0$	A1	
	(ii)	$\cos x(\sqrt{3}\sin x + \cos x) = 0$	M1	
		$\cos x = 0$ or $\tan x = -\frac{1}{\sqrt{3}}$	M1	
		$x = \frac{\pi}{2}, \frac{5\pi}{6}$	A2	**(9)**

6.	*(i)*	x	0	0.5	1	1.5	2	2.5	3	
		y	0	0.5774	0.7071	0.7746	0.8165	0.8452	0.8660	M1 A1

area $\approx \frac{1}{3}\times 0.5\times[0+0.8660+4(0.5774+0.7746+0.8452)+2(0.7071+0.8165)]$ M1

$= 2.12$ (3sf) A1

(ii) $= \pi\int_0^3 \frac{x}{x+1}\, dx$ M1

$= \pi\int_0^3 \frac{x+1-1}{x+1}\, dx = \pi\int_0^3 (1 - \frac{1}{x+1})\, dx$ M1

$= \pi[x - \ln|x+1|]_0^3$ M1 A1

$= \pi\{(3 - \ln 4) - (0)\} = \pi(3 - \ln 4)$ M1 A1 **(10)**

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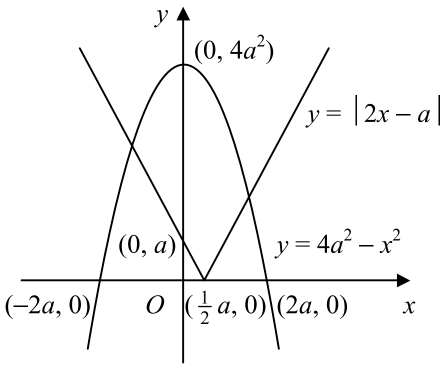
7. *(i)*



B3

B2

(ii) $4 - x^2 = 2x - 1$ M1

$x^2 + 2x - 5 = 0$

$x = \frac{-2\pm\sqrt{4+20}}{2} = \frac{-2\pm 2\sqrt{6}}{2}$ M1

$x > \frac{1}{2}\ \therefore x = -1 + \sqrt{6}$ A1

$4 - x^2 = -(2x - 1)$ M1

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$ M1

$x < \frac{1}{2}\ \therefore x = -1,\qquad x = -1, -1 + \sqrt{6}$ A1 **(11)**

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8. *(i)* $\frac{dy}{dx} = -e^2x^{-2} + e^x$ M1 A1

(ii) SP: $-e^2x^{-2} + e^x = 0$ M1

let $f(x) = -e^2x^{-2} + e^x$

$f(1.3) = -0.70,\ f(1.4) = 0.29$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

(iii) $x = 2,\ y = \frac{3}{2}e^2,\ \text{grad} = \frac{3}{4}e^2$ M1

$\therefore\ y - \frac{3}{2}e^2 = \frac{3}{4}e^2(x - 2)$ M1 A1

$y = \frac{3}{4}e^2x$

$\therefore\ x = 0 \Rightarrow y = 0$ so passes through origin A1

(iv) $x_1 = -1.125589,\ x_2 = -1.125803,\ x_3 = -1.125804$ (7sf) M1 A1

$\therefore$ x-coordinate of $B = -1.1258$ (5sf) A1 **(12)**

Total **(72)**

Performance Record – C3 Paper L

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	differentiation	exponentials and logarithms	functions	trigonometry	trigonometry	Simpson's rule, integration	functions	differentiation, numerical methods	
Marks	6	7	8	9	9	10	11	12	72
Student									