FOR OCR

GCE Examinations

Advanced / Advanced Subsidiary

Core Mathematics C4

Paper E

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

Written by Shaun Armstrong

1. $$f(x) = 1 + \frac{4x}{2x-5} - \frac{15}{2x^2-7x+5}.$$

Show that

$$f(x) = \frac{3x+2}{x-1}.$$ [4]

2. A curve has the equation

$$x^2 - 3xy - y^2 = 12.$$

(i) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [4]

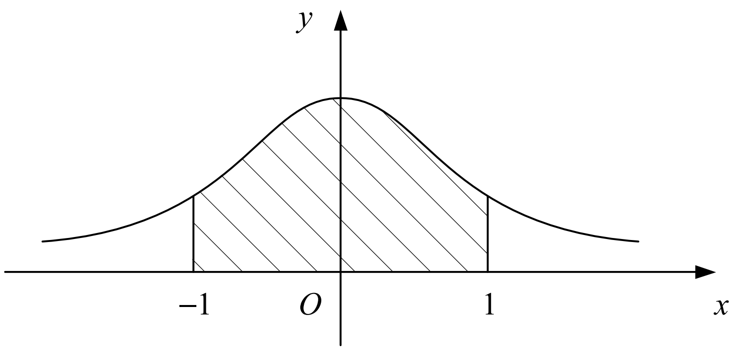
(ii) Find an equation for the tangent to the curve at the point (2, –2). [3]

3. Find

(i) $\int \frac{x}{2-x^2}\, dx$, [3]

(ii) $\int x^2 e^{-x}\, dx$. [5]

4.



The diagram shows the curve with parametric equations

$$x = \tan\theta,\quad y = \cos^2\theta,\quad -\tfrac{\pi}{2} < \theta < \tfrac{\pi}{2}.$$

(i) Find a cartesian equation for the curve. [3]

The shaded region is bounded by the curve, the x-axis and the lines $x = -1$ and $x = 1$.

(ii) Using integration, with the substitution $x = \tan u$, find the area of the shaded region. [5]

5. *(i)* Expand $(4 - x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2, simplifying each coefficient. [4]

(ii) State the set of values of x for which your expansion is valid. [1]

(iii) Use your expansion with $x = 0.01$ to find the value of $\sqrt{399}$, giving your answer to 9 significant figures. [4]

6. *(i)* Use the derivative of $\cos x$ to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x.$$ [4]

The curve C has the equation $y = \mathrm{e}^{2x} \sec x, \; -\frac{\pi}{2} < x < \frac{\pi}{2}$.

(ii) Find an equation for the tangent to C at the point where it crosses the y-axis. [4]

(iii) Find, to 2 decimal places, the x-coordinate of the stationary point of C. [3]

7. The line l_1 passes through the points A and B with position vectors $(3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$ and $(8\mathbf{j} - 6\mathbf{k})$ respectively, relative to a fixed origin.

(i) Find a vector equation for l_1. [2]

The line l_2 has vector equation

$$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) + \mu(7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}),$$

where μ is a scalar parameter.

(ii) Show that lines l_1 and l_2 intersect. [4]

(iii) Find the coordinates of the point where l_1 and l_2 intersect. [1]

The point C lies on l_2 and is such that AC is perpendicular to AB.

(iv) Find the position vector of C. [5]

Turn over

8. When a plague of locusts attacks a wheat crop, the proportion of the crop destroyed after t hours is denoted by x. In a model, it is assumed that the rate at which the crop is destroyed is proportional to $x(1 - x)$.

A plague of locusts is discovered in a wheat crop when one quarter of the crop has been destroyed.

Given that the rate of destruction at this instant is such that if it remained constant, the crop would be completely destroyed in a further six hours,

(i) show that $\frac{dx}{dt} = \frac{2}{3}x(1 - x)$, [3]

(ii) find the percentage of the crop destroyed three hours after the plague of locusts is first discovered. [10]