OCR
Oxford Cambridge and RSA

Thursday 21 October 2021 – Afternoon

AS Level Further Mathematics A

Y535/01 Additional Pure Mathematics

Time allowed: 1 hour 15 minutes

9054624919

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for AS Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The points A, B and C have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ respectively, relative to the origin O.

(a) (i) Calculate $\mathbf{a} \times \mathbf{b}$, giving your answer as a multiple of $\mathbf{c}$. **[1]**

(ii) Explain, geometrically, why $\mathbf{a} \times \mathbf{b}$ must be a multiple of $\mathbf{c}$. **[1]**

(b) Use a vector product method to calculate the area of triangle ABC. **[3]**

2 The surface S is given by $z = x^2 + 4xy$ for $-6 \leqslant x \leqslant 6$ and $-2 \leqslant y \leqslant 2$.

(a) (i) Write down the equation of any **one** section of S which is parallel to the x-z plane. **[1]**

(ii) Sketch the section of **(a)(i)** on the axes provided in the Printed Answer Booklet. **[2]**

(b) Write down the equation of any **one** contour of S which does **not** include the origin. **[1]**

3 For positive integers n, the sequence of Fibonacci numbers, $\{F_n\}$, starts with the terms $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, … and is given by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ $(n \geqslant 3)$.

(a) Show that $F_{3k+3} = 2F_{3k+1} + F_{3k}$, where k is a positive integer. **[2]**

(b) Prove by induction that F_{3n} is even for all positive integers n. **[4]**

4 (a) Let $a = 1071$ and $b = 67$.

(i) Find the unique integers q and r such that $a = bq + r$, where $q > 0$ and $0 \leqslant r < b$. **[1]**

(ii) Hence express the answer to **(a)(i)** in the form of a linear congruence modulo b. **[1]**

(b) Use the fact that $358 \times 715 - 239 \times 1071 = 1$ to prove that 715 and 1071 are co-prime. **[4]**

5 A trading company deals in two goods. The formula used to estimate z, the total weekly cost to the company of trading the two goods, in tens of thousands of pounds, is

$$z = 0.9x + \frac{0.096y}{x} - x^2y^2,$$

where x and y are the masses, in thousands of tonnes, of the two goods.

You are given that $x > 0$ and $y > 0$.

(a) In the first week of trading, it was found that the values of x and y corresponded to the stationary value of z.

Determine the total cost to the company for this week. **[7]**

(b) For the second week, the company intends to make a small change in either x or y in order to reduce the total weekly cost.

Determine whether the company should change x or y. (You are not expected to say by how much the company should reduce its costs.) **[4]**

6 The set S consists of the following four complex numbers.

$$\sqrt{3}+\mathrm{i} \qquad -\sqrt{3}-\mathrm{i} \qquad 1-\mathrm{i}\sqrt{3} \qquad -1+\mathrm{i}\sqrt{3}$$

For $z_1, z_2 \in S$, the binary operation $\bigcirc$ is defined by $z_1 \bigcirc z_2 = \frac{1}{4}(1+\mathrm{i}\sqrt{3})z_1z_2$.

(a) (i) Complete the Cayley table for $(S, \bigcirc)$ given in the Printed Answer Booklet. **[3]**

(ii) Verify that $(S, \bigcirc)$ is a group. **[4]**

(iii) State the order of each element of $(S, \bigcirc)$. **[1]**

(b) Write down the only proper subgroup of $(S, \bigcirc)$. **[1]**

(c) (i) Explain why $(S, \bigcirc)$ is a cyclic group. **[1]**

(ii) List all possible generators of $(S, \bigcirc)$. **[1]**

Turn over for questions 7 and 8

7 **(a)** Let $f(n) = 2^{4n+3} + 3^{3n+1}$.

Use arithmetic modulo 11 to prove that $f(n) \equiv 0 \pmod{11}$ for all integers $n \geqslant 0$. **[4]**

(b) Use the standard test for divisibility by 11 to prove the following statements.

(i) $10^{33} + 1$ is divisible by 11 **[2]**

(ii) $10^{33} + 1$ is divisible by 121 **[4]**

8 A sequence $\{u_n\}$ is defined by the recurrence system

$u_1 = 1$ and $u_{n+1} = a - \dfrac{a^2}{2u_n}$ for $n \geqslant 1$, where a is a positive constant.

Determine with justification the behaviour of the sequence for all possible values of a. **[7]**

END OF QUESTION PAPER