Pearson Edexcel

Mark Scheme (Result)

October 2020

Pearson Edexcel GCE Further Mathematics

Advanced Level in

Further Mathematics Paper 1 9FM0/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

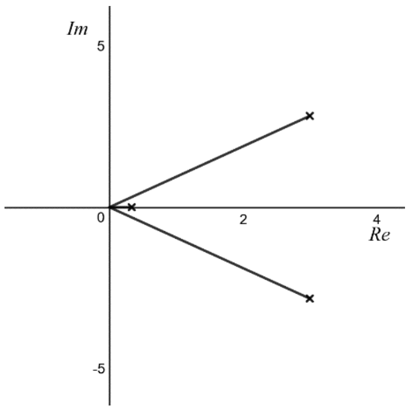
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(a)	$\beta = 3+2\sqrt{2}\,\mathrm{i}$ is also a root	B1	1.2
	$\alpha\beta = 17, \alpha+\beta = 6$	B1	1.1b
	$\alpha\beta+\alpha\gamma+\beta\gamma = \dfrac{57}{3}$	M1	1.1b
	$\alpha\gamma+\beta\gamma = \dfrac{57}{3}-17 = \gamma(\alpha+\beta) = 6\gamma \Rightarrow \gamma = \ldots$	M1	3.1a
	$\gamma = \dfrac{1}{3}$	A1	2.2a
		B1	1.1b
		B1ft	1.1b
		(7)	
	(a) Alternative:		
	$\beta = 3+2\sqrt{2}\,\mathrm{i}$ is also a root	B1	1.2
	$\left(z-\left(3+2\sqrt{2}\,\mathrm{i}\right)\right)\left(z-\left(3-2\sqrt{2}\,\mathrm{i}\right)\right) = z^2-6z+17$	B1	1.1b
	$\mathrm{f}(z) = \left(z^2-6z+17\right)(3z+a) = 3z^3+az^2-18z^2-6az+51z+17a$	M1	1.1b
	$\Rightarrow 51-6a = 57 \Rightarrow a = -1 \Rightarrow \gamma = \ldots$	M1	3.1a
	$\gamma = \dfrac{1}{3}$	A1	2.2a
	Then B1 B1ft as above		
		(7)	
(b)	$3-2\sqrt{2}\,\mathrm{i}+3+2\sqrt{2}\,\mathrm{i}+\dfrac{1}{3} = -\dfrac{p}{3} \Rightarrow p = \ldots$ or $\left(3-2\sqrt{2}\,\mathrm{i}\right)\left(3+2\sqrt{2}\,\mathrm{i}\right)\times\dfrac{1}{3} = -\dfrac{q}{3} \Rightarrow q = \ldots$	M1	3.1a
	$p = -19$ **or** $q = -17$	A1	1.1b
	$p = -19$ **and** $q = -17$	A1	1.1b
		(3)	
	(b) Alternative:		
	$\mathrm{f}(z) = \left(z^2-6z+17\right)(3z-1) = 3z^3+pz^2+57z+q$	M1	3.1a

	$\Rightarrow p = ..., q = ...$		
	$p = -19$ **or** $q = -17$	A1	1.1b
	$p = -19$ **and** $q = -17$	A1	1.1b
		(3)	
			(10 marks)

Notes

(a)

B1: Identifies the correct complex conjugate as another root

B1: Correct values for the sum and product for the conjugate pair

M1: Correct application of the pair sum

M1: Identifies a complete and correct strategy for identifying the third root

A1: Deduces the correct third root

B1: $3 \pm 2\sqrt{2}\,\mathrm{i}$ plotted correctly, in quadrants 1 and 4 which are reflections in the real axis. Do not be concerned about labelling or scaling.

B1ft: Their real root plotted correctly, in correct relative position to the two complex roots. Scales are not needed but if correct, the real root must be close to the origin compared to the complex roots.

Alternative:

B1: Identifies the correct complex conjugate as another root

B1: Correct quadratic factor obtained

M1: Expands their quadratic×(3*z* + "*a*") or attempts to factor out the quadratic, or use long division, leading to a factor (3*z* + "*a*"). Implied by seeing $\left(z^2 - 6z + 17\right)\left(3z + a\right)$ with any value of *a* (or with their quadratic).

M1: Proceeds to extract the root from their third factor of from (3*z* + "*a*").

A1: Deduces the correct third root. If not explicitly stated, look for it on their diagram.

B1: $3 \pm 2\sqrt{2}\,\mathrm{i}$ plotted correctly, as above

B1ft: Their real root plotted correctly as above.

(b)

M1: Correct strategy used for identifying at least one of *p* or *q*

A1: At least one value correct

A1: Both values correct

Alternative:

M1: Correct strategy by expanding their quadratic and linear factors to identifying at least one of *p* or *q*

A1: At least one value correct

A1: Both values correct

Note: some may attempt to use the factor theorem with the complex root.

$\mathrm{f}\left(3 - 2\mathrm{i}\sqrt{2}\right) = 36 + p + q + \mathrm{i}\left(-228\sqrt{2} - 12\sqrt{2}p\right) = 0$

2nd B1: equate real and imaginary components to 0 to get correct equations

$36 + p + q = 0, -228\sqrt{2} - 12\sqrt{2}p = 0$

1st M1: solves their equations $\Rightarrow p = -19, q = -17$

2nd M1: Solves the cubic (may be from calculator). The 1st B1 may then be implied for the second complex root, and the rest as main scheme.

Question	Scheme	Marks	AOs
2(a)	E.g. • Because the interval being integrated over is unbounded • Accept because the upper limit is infinity • Accept because a limit is required to evaluate it	B1	2.4
		(1)	
(b)	$\frac{1}{x(2x+5)} = \frac{A}{x} + \frac{B}{2x+5} \Rightarrow A = \ldots, B = \ldots$	M1	3.1a
	$\frac{1}{x(2x+5)} = \frac{1}{5x} - \frac{2}{5(2x+5)}$	A1	1.1b
	$\int \frac{1}{5x} - \frac{2}{5(2x+5)}\,dx = \frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5)$	A1ft	1.1b
	$\frac{1}{5}\ln x - \frac{1}{5}\ln(2x+5) = \frac{1}{5}\ln\frac{x}{(2x+5)}$	M1	2.1
	$\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{x}{2x+5}\right\} = \frac{1}{5}\ln\frac{1}{2}$	B1	2.2a
	$\Rightarrow \int_1^{\infty} \frac{1}{x(2x+5)}\,dx = \frac{1}{5}\ln\frac{1}{2} - \frac{1}{5}\ln\frac{1}{7} = \frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	
			(7 marks)

Notes

(a)

B1: For a suitable explanation with no contrary reasoning. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity". Do not award if there are erroneous statements e.g. referring to as $x = 0$ the integrand is not defined. Do not accept "because one of the limits is undefined" unless they state they mean ∞. Do not accept "it is undefined when $x = \infty$" without reference to "it" being the upper limit.

(b)

M1: Selects the correct form for partial fractions and proceeds to find values for *A* and *B*

A1: Correct constants or partial fractions

A1ft: $\int \frac{p}{x} + \frac{q}{2x+5}\,dx = p\ln x + \frac{q}{2}\ln(2x+5)$ Note that $\frac{1}{5}\ln 5x - \frac{1}{5}\ln(10x+25)$ is correct.

M1: Combines logs correctly. May see $-\frac{1}{5}\ln\left(\frac{2x+5}{x}\right) = -\frac{1}{5}\ln\left(2+\frac{5}{x}\right)$

B1: Correct upper limit for $x \to \infty$ by recognising the dominant terms. (Simply replacing *x* with ∞ scores B0)

A1: Deduces the correct value for the improper integral in the correct form

Question	Scheme	Marks	AOs
(b) **Way 2**	$\dfrac{1}{x(2x+5)}=\dfrac{1}{2\left(x^2+\dfrac{5}{2}x\right)}=\dfrac{1}{2}\times\dfrac{1}{\left(x+\dfrac{5}{4}\right)^2-\dfrac{25}{16}}$	M1 A1	3.1a 1.1b
	$\displaystyle\int\frac{1}{x(2x+5)}\,\mathrm{d}x=\frac{1}{2}\times\frac{2}{5}\ln\left\lvert\frac{x+\frac{5}{4}-\frac{5}{4}}{x+\frac{5}{4}+\frac{5}{4}}\right\rvert=\frac{1}{5}\ln\left\lvert\frac{2x}{2x+5}\right\rvert$	M1 A1ft	2.1 1.1b
	$\displaystyle\lim_{x\to\infty}\left\{\frac{1}{5}\ln\frac{2x}{2x+5}\right\}=\frac{1}{5}\ln\frac{2}{2}=0$	B1	2.2a
	$\displaystyle\Rightarrow\int_1^{\infty}\frac{1}{x(2x+5)}\,\mathrm{d}x=0-\frac{1}{5}\ln\frac{2}{7}=\frac{1}{5}\ln\frac{7}{2}$	A1	1.1b
		(6)	

Notes

Note the method marks as MAMABA, and should be entered in this order on ePEN.

M1: Expands the denominator and completes the square.

A1: Correct expression

M1: For $\dfrac{1}{(x+p)^2-a^2}\to k\ln\left\lvert\dfrac{x+p-a}{x+p+a}\right\rvert$

A1ft: $\dfrac{1}{2}\dfrac{1}{(x+a)^2-a^2}\to\dfrac{1}{2a}\ln\left\lvert\dfrac{x}{x+2a}\right\rvert$ with their a (may be simplified as in scheme).

B1: Correct upper limit for $x\to\infty$ by recognising the dominant terms. (Simply replacing x with ∞ scores B0) Note in this method the upper limit evaluates to zero.

A1: Deduces the correct value for the improper integral in the correct form. Accept $-\dfrac{1}{5}\ln\dfrac{2}{7}$

Question	Scheme	Marks	AOs
3	$3(1-\sin\theta)=1+\sin\theta \Rightarrow \sin\theta=\dfrac{1}{2} \Rightarrow \theta=...$	M1	3.1a
	$\theta=\dfrac{\pi}{6}\left(\text{or}\ \dfrac{5\pi}{6}\right)$	A1	1.1b
	Use of $\dfrac{1}{2}\int(1+\sin\theta)^2\,\mathrm{d}\theta$ or $\dfrac{1}{2}\int\{3(1-\sin\theta)\}^2\,\mathrm{d}\theta$	M1	1.1a
	$\left(\dfrac{1}{2}\right)\int\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta$ $=\left(\dfrac{1}{2}\right)\int\left[1+2\sin\theta+\sin^2\theta-9+18\sin\theta-9\sin^2\theta\right]\mathrm{d}\theta$ or $\int(1+\sin\theta)^2\,\mathrm{d}\theta=\int(1+2\sin\theta+\sin^2\theta)\,\mathrm{d}\theta$ **and** $\int 9(1-\sin\theta)^2\,\mathrm{d}\theta=9\int(1-2\sin\theta+\sin^2\theta)\,\mathrm{d}\theta$	M1 A1	2.1 1.1b
	$\int\sin^2\theta\,\mathrm{d}\theta=\dfrac{1}{2}\int(1-\cos2\theta)\,\mathrm{d}\theta \Rightarrow$ $\int\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta=2\sin2\theta-12\theta-20\cos\theta$	M1 A1	3.1a 1.1b
	$A=\dfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta$ or $A=2\times\dfrac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta$ $=\dfrac{1}{2}\left\{\left(-\sqrt{3}-10\pi+10\sqrt{3}\right)-\left(\sqrt{3}-2\pi-10\sqrt{3}\right)\right\}=...$	DM1	3.1a
	$=9\sqrt{3}-4\pi$	A1	1.1b
		(9)	
			(9 marks)

Notes

M1: Realises that the angles at the intersection are required and solves $C_1 = C_2$ to obtain a value for θ

A1: Correct value for θ. Must be in radians – if given in degrees you may need to check later to see if they convert to radians before substitution.

M1: Evidence selecting the correct polar area formula on either curve

M1: Fully expands both expressions for r^2 either as parts of separate integrals or as one complete integral. (Can be scored from incorrect polar area formula, e.g. missing the ½)

A1: Correct expansions for both curves (may be unsimplified)

M1: Selects the correct strategy by applying the correct double angle identity in order to reach an integrable form and attempting the integration of at least one of the curves.
A1: Correct integration (of both integrals if done separately),
FYI: If done separately the correct integrals are
$\int(1+\sin\theta)^2\,\mathrm{d}\theta=\theta-2\cos\theta+\frac{1}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)=\frac{3}{2}\theta-2\cos\theta-\frac{1}{4}\sin 2\theta$ and
$\int 9(1-\sin\theta)^2\,\mathrm{d}\theta=9\theta+18\cos\theta+\frac{9}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)=\frac{27}{2}\theta+18\cos\theta-\frac{9}{4}\sin 2\theta$
DM1: Depends on all previous M's. For a fully correct strategy with appropriate limits correctly applied to their integral or integrals and terms combined if necessary. Make sure that if limits of $\frac{\pi}{6}$ and $\frac{\pi}{2}$ are used that the area is doubled as part of the strategy.
A1: Correct area

Question	Scheme	Marks	AOs
4(a)	Attempts normal vector: E.g. let $\mathbf{n}=a\mathbf{i}+b\mathbf{j}+\mathbf{k}$ then $a+2b-3=0,\ -a+2b+1=0$ $\Rightarrow a=...,b=...$ or $\mathbf{n}=(\mathbf{i}+2\mathbf{j}-3\mathbf{k})\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k})$	M1	3.1a
	$\mathbf{n}=k(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$	A1	1.1b
	$(4\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}+4\mathbf{j}-\mathbf{k})=...$	M1	1.1b
	$4x+y+2z=10$	A1	2.5
		(4)	
	Alternative:		
	$x=2+\lambda-\mu$, $y=4+2\lambda+2\mu$, $z=-1-3\lambda+\mu$ $\Rightarrow$ $2x+y=8+4\lambda$, $y-2z=6+8\lambda$	M1 A1	3.1a 1.1b
	$2(2x+y-8)=y-2z-6$ $(4x+y+2z=10)$	M1 A1	1.1b 2.5
		(4)	
(b)	$\dfrac{x-1}{5}=\dfrac{y-3}{-3}=\dfrac{z+2}{4}\Rightarrow\mathbf{r}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(5\mathbf{i}-3\mathbf{j}+4\mathbf{k})$ $4(1+5\lambda)+3-3\lambda+2(4\lambda-2)=10\Rightarrow\lambda=...$	M1	3.1a
	$\lambda=\dfrac{7}{25}\Rightarrow\mathbf{r}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\dfrac{7}{25}(5\mathbf{i}-3\mathbf{j}+4\mathbf{k})$	dM1	1.1b
	$\left(\dfrac{12}{5},\dfrac{54}{25},-\dfrac{22}{25}\right)$	A1	1.1b
		(3)	
	Alternative: $4x+\left(-\dfrac{3}{5}(x-1)+3\right)+2\left(\dfrac{4}{5}(x-1)-2\right)=10\Rightarrow x=...$	M1	3.1a
	$\Rightarrow y=...,\ z=...$	M1	1.1b
	$\left(\dfrac{12}{5},\dfrac{54}{25},-\dfrac{22}{25}\right)$	A1	1.1b
		(3)	
(c)	$(4\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}-\mathbf{j}+3\mathbf{k})=8-1+6=13$ $13=\sqrt{14}\sqrt{21}\cos\theta\Rightarrow\theta=...$	M1	1.1b
	$\theta=41°$	A1	1.1b
		(2)	
			(9 marks)

Notes
Accept equivalent vector notation throughout. (a) M1: Starts by attempting to find a normal vector using a correct method. Allow if there are sign errors in attempts at the cross product. A1: Obtains a correct normal vector M1: Attempts scalar product between their normal and a point in the plane A1: Correct Cartesian form (accept any equivalent Cartesian equation) Alternative M1: Uses the component form to eliminate one of the scalar parameters A1: Two correct equations with one parameter eliminated OR a correct equation for each parameter in terms of *x*, *y* and *z* M1: Forms a Cartesian equation A1: Correct Cartesian equation (accept any equivalent form) (b) M1: Interprets the Cartesian form to give a parametric form (allow sign slips) and substitutes this into their Cartesian equation and proceeds to find a value for their parameter. NB: Attempts at $\begin{pmatrix} 2+\lambda-\mu \\ 4+2\lambda+2\mu \\ -1-3\lambda+\mu \end{pmatrix} = \begin{pmatrix} 1+5\lambda \\ 3-3\lambda \\ -2+4\lambda \end{pmatrix}$ will score M0 as there are only two parameters, but $\begin{pmatrix} 2+\lambda-\mu \\ 4+2\lambda+2\mu \\ -1-3\lambda+\mu \end{pmatrix} = \begin{pmatrix} 1+5\gamma \\ 3-3\gamma \\ -2+4\gamma \end{pmatrix}$ leading to a value for γ from solving three equations in three unknowns in M1. dM1: Substitutes their parameter value back into the parametric form of the line. The parameter must have come from a correct attempt to find the value at intersection. A1: Correct coordinates. Accept as *x* = …, *y*=, … *z* = … or as a vector. Alternative: M1: Eliminates two of the variables from the equation of plane using the Cartesian equation of the line and solves the linear equation. dM1: Finds the other two coordinates. A1: Correct coordinates, as above. (c) M1: Complete and correct scalar product method leading to a value for θ. Note that if sinθ is used instead of cosθ then they must also apply 90 – θ to access the method. A1: Correct angle, accept awrt 41. as their final answer (do not isw if they go on to give e.g. $(180-41)°$)

Question	Scheme	Marks	AOs
5(a)	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-5\frac{\mathrm{d}x}{\mathrm{d}t}+10\frac{\mathrm{d}y}{\mathrm{d}t}$ oe e.g. $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{10}\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}\right)$	B1	1.1b
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=-5\frac{\mathrm{d}x}{\mathrm{d}t}+10\left(-2x+3y-4\right)$ $=-5\frac{\mathrm{d}x}{\mathrm{d}t}-20x+\frac{30}{10}\left(\frac{\mathrm{d}x}{\mathrm{d}t}+5x+30\right)-40$ Or $\frac{1}{10}\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+5\frac{\mathrm{d}x}{\mathrm{d}t}\right)=-2x+\frac{3}{10}\left(30+5x+\frac{\mathrm{d}x}{\mathrm{d}t}\right)-4$	M1	2.1
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}+5x=50*$	A1*	1.1b
		(3)	
(b)	$m^2+2m+5=0\Rightarrow m=...$	M1	3.4
	$m=-1\pm 2\mathrm{i}$	A1	1.1b
	$m=\alpha\pm\beta\mathrm{i}\Rightarrow x=\mathrm{e}^{\alpha t}\left(A\cos\beta t+B\sin\beta t\right)=...$	M1	3.4
	$x=\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)$	A1	1.1b
	PI: Try $x=k\Rightarrow 5k=50\Rightarrow k=10$	M1	3.4
	$GS: x=\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)+10$	A1ft	1.1b
		(6)	
(c)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{e}^{-t}\left(2B\cos 2t-2A\sin 2t\right)-\mathrm{e}^{-t}\left(A\cos 2t+B\sin 2t\right)$	B1ft	1.1b
	$(y=)\frac{1}{10}\left(\frac{\mathrm{d}x}{\mathrm{d}t}+5x+30\right)=...$	M1	3.4
	$y=\frac{1}{10}\mathrm{e}^{-t}\left((4A+2B)\cos 2t+(4B-2A)\sin 2t\right)+8$	A1	1.1b
		(3)	
(d)	$t=0, x=2\Rightarrow 2=A+10\Rightarrow A=-8$	M1	3.1b
	$t=0, y=5\Rightarrow 5=\frac{1}{10}\left(2B-32\right)+8\Rightarrow B=1$	M1	3.3
	$x=\mathrm{e}^{-t}\left(\sin 2t-8\cos 2t\right)+10$	A1	2.2a
	$y=\mathrm{e}^{-t}\left(2\sin 2t-3\cos 2t\right)+8$	A1	2.2a
		(4)	
(e)	E.g When $t>8$, the amount of compound X and the amount of compound Y remain (approximately) constant at 10 and 8 respectively, which suggests that the chemical reaction has stopped. This supports the scientist's claim.	B1	3.5a
		(1)	
			(17 marks)

Notes
(a) B1: Differentiates the first equation with respect to t correctly. May have rearranged to make y the subject first. The dot notation for derivatives may be used. M1: Uses the second equation to eliminate y to achieve an equation in x, $\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$. A1*: Achieves the printed answer with no errors. (b) M1: Uses the model to form and attempts to attempts to solve the auxiliary equation (Accept a correct equation followed by two values for m as an attempt to solve.) A1: Correct roots of the AE M1: Uses the model to form the complementary function. Must be in terms of t only (not x) A1: Correct CF M1: Chooses the correct form of the PI according to the model and uses a complete method to find the PI A1ft: Combines their CF (which need not be correct) with the correct PI to give x in terms of t so look for x = their CF + 10 (c) B1ft: Correct differentiation of their x. Follow through their $\mathrm{e}^{\alpha t}\left(A\cos\beta t + B\sin\beta t\right)$ M1: Uses the model and their answer to part (b) to find an expression for y in terms of t A1: Correct equation. Mark the final answer but there is not need for terms to be gathered but must have $y = \ldots$. (d) M1: Realises the need to use the initial conditions in the equation for x M1: Realises the need to use the initial conditions in the equation for y to find both unknown constants - must have equations from which both unknowns can be found. Alternatively, a complete method using $\frac{\mathrm{d}x}{\mathrm{d}t}$ to find the second constant is made. A1: Deduces the correct equation for x A1: Deduces the correct equation for y. For this equation constants should have been gathered. (e) B1: Allow for any appropriate comment with valid supporting reason. They must have equations of the correct form from (d). The coefficients may be incorrect, but they must have positive limits for each of x and y. Both x and y should be considered (see below for exception), and a reason and some comment about the suitability of the model made (though you may allow implicit conclusions). E.g. • for values of $t > 8$, the amounts of compounds X and Y present settle at 10 and 8 without really varying, which supports the claim. • $\frac{\mathrm{d}x}{\mathrm{d}t} \approx 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} \approx 0$ when $t = 8$, so neither are changing, which supports the claim. • As t gets large x and y tend to limits to 10 and 8 neither will be zero, hence the claim is not supported. • $x = 10.0$ (awrt) and $y = 8.00$ (awrt) when $t = 8$, since neither is zero it is likely the reaction is still continuing so the claim is not supported. Exception: Allow a reason that states the model assumes that the reaction continues indefinitely, so the claim is not supported. (The reaction stopping would require a change in the model.)

Do NOT allow an answer that only considers x or y. E.g. $x = 10$ when $t = 8$ so the model is not supported is B0 since there is no consideration that y may be zero and hence end the reaction.

Alt for (c) and (d) restarting:

B1: Correct second order equation for y formed: $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y}{\mathrm{d}t} + 5y = 40$

M1: Full method to obtain the general solution: they may recognise the similarity to the equation in x and jump straight to finding the PI, or may form the aux equation etc again, but look for an attempt that combines a (correctly formed) CF and a PI. For this mark allow if the constants used are the same as those for the equation in x.

A1: Correct solution for y with different constants than those for x, though allow recovery if they realise in (d) that they need different constants.

For (d)

M1: As main scheme, allow for using the initial conditions in one equation to make a start finding the constants.

M1: For a full method to obtain all four constants – if the same constant were used for both equations in (c) (inconsistently) then this mark cannot be scored. A full method here would, for instance, require finding $\frac{\mathrm{d}x}{\mathrm{d}t}$ and using this along with the given initial equations and initial conditions to find the second constants for each equation.

A1: One correct equation with SC of being qualified by the first M only if a full method to find both constants for just one equation is made (so M1M0A1A0 is possible in this case).

A1: Both equations correct.

Question	Scheme	Marks	AOs
6(i)	When n = 1, $\sum_{r=1}^{1}(3r+1)(r+2)=4\times3=12$ $1(1+2)(1+3)=12$ (so the statement is true for n = 1)	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^{k}(3r+1)(r+2)=k(k+2)(k+3)$	M1	2.4
	$\sum_{r=1}^{k+1}(3r+1)(r+2)=k(k+2)(k+3)+(3k+4)(k+3)$	M1	2.1
	$=(k+3)(k^2+5k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1}(3r+1)(r+2)=(k+1)(k+3)(k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1}(3r+1)(r+2)=(\underline{k+1})(\underline{k+1}+2)(\underline{k+1}+3)$ If the statement is <u>true for *n* = *k* then</u> it has been shown <u>true for *n* = *k* + 1</u> and as it is <u>true for *n* = 1</u>, the statement <u>is true for all (positive integers) *n*.</u>	A1	2.4
		(6)	
(ii) Way 1	When n = 1, $4^1+5^1+6^1=15$ so the statement is true for n = 1	B1	2.2a
	Assume true for $n = k$ so $4^k+5^k+6^k$ is divisible by 15	M1	2.4
	$\text{f}(k+2)=4^{k+2}+5^{k+2}+6^{k+2}$	M1	2.1
	$=16\times4^k+16\times5^k+16\times6^k+9\times5^k+20\times6^k$ $=16\text{f}(k)+45\times5^{k-1}+120\times6^{k-1}$	A1 A1	1.1b 1.1b
	E.g As 15 divides f(k), 45 and 120, so 15 divides f(k+1). <u>If true for *n* = *k*</u> then <u>true for *n* = *k* + 2</u>, <u>true for *n* = 1</u> so <u>true for all positive odd integers *n*</u>	A1	2.4
		(6)	
(ii) Way 2	When n = 1, $4^1+5^1+6^1=15$ so the statement is true for n = 1	B1	2.2a
	Assume true for $n = k$ so $4^k+5^k+6^k$ is divisible by 15	M1	2.4
	$\text{f}(k+2)-\text{f}(k)=4^{k+2}+5^{k+2}+6^{k+2}-4^k-5^k-6^k$	M1	2.1
	$=15\times4^k+24\times5^k+35\times6^k$ $=15\text{f}(k)+45\times5^{k-1}+120\times6^{k-1}$	A1	1.1b
	$\text{f}(k+2)=16\text{f}(k)+45\times5^{k-1}+120\times6^{k-1}$	A1	1.1b
	E.g $\text{f}(k+2)=16\text{f}(k)+15(3\times5^{k-1}+8\times6^{k-1})$ so <u>if true for *n* = *k*</u> then <u>true for *n* = *k* + 2</u>, <u>true for *n* = 1</u> so <u>true for all positive odd integers *n*</u>	A1	2.4
		(6)	

(ii) Way 3	When $n = 1$, $4^1 + 5^1 + 6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = 2k+1$ so f($2k$+1) = $4^{2k+1} + 5^{2k+1} + 6^{2k+1}$ is divisible by 15	M1	2.4
	$\mathrm{f}\left(2k+3\right) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3}$ or $\mathrm{f}\left(2k+3\right) - \mathrm{f}\left(2k+1\right) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3} - 4^{2k+1} - 5^{2k+1} - 6^{2k+1}$	M1	2.1
	$\mathrm{f}(2k+3) = 16\times 4^{2k+1} + 25\times 5^{2k+1} + 36\times 6^{2k+1}$ $= 16\left(4^{2k+1} + 5^{2k+1} + 6^{2k+1}\right) + 9\times 5^{2k+1} + 20\times 6^{2k+1}$ OR $\mathrm{f}(2k+3) - \mathrm{f}(2k+1) = 16\times 4^{2k+1} + 25\times 5^{2k+1} + 36\times 6^{2k+1} - 4^{2k+1} - 5^{2k+1} - 6^{2k+1}$ $= 15\times 4^{2k+1} + 120\times 5^{2k} + 210\times 6^{2k}$	A1	1.1b
	$\mathrm{f}(2k+3) = 16\left(4^{2k+1} + 5^{2k+1} + 6^{2k+1}\right) + 45\times 5^{2k} + 120\times 6^{2k}$ OR $\mathrm{f}\left(2k+3\right) = \mathrm{f}\left(2k+1\right) + 15\times 4^{2k+1} + 120\times 5^{2k} + 210\times 6^{2k}$	A1	1.1b
	$\mathrm{f}(2k+3) = 16\left(4^{2k+1} + 5^{2k+1} + 6^{2k+1}\right) + 15\left(3\times 5^{2k} + 8\times 6^{2k}\right)$ OR $\mathrm{f}\left(2k+3\right) = \mathrm{f}\left(2k+1\right) + 15\left(4^{2k+1} + 8\times 5^{2k} + 14\times 6^{2k}\right)$ and If true for $n = 2k+1$ then true for $n = 2k+3$, true for $n = 1$ so true for all positive odd integers n	A1	2.4
		(6)	
			(12 marks)

Notes

(i)
B1: Shows the statement is true for $n = 1$ by evaluating **both** sides. There is no need for statement "hence true for n =1" for this mark but if they never state this the final A will be forfeited.
Look for a minimum of $4\times 3 = 12$ for the LHS and $1\times 3\times 4$ for the RHS. If only one side is explicitly evaluated, it is B0, but all other marks may be gained.
M1: Makes an assumption statement that assumes the result is true for $n = k$
M1: Makes the inductive step by attempting to add the $(k + 1)^{\text{th}}$ term to the assumed result. Attempts at using the standard summation formulae score M0, as the question requires induction.
A1: Correct expression with at least one correct linear factor
A1: Obtains a fully correct factorised expression. May be as in scheme or in terms of $k + 1$.
A1: Correct complete conclusion with all ideas conveyed at the end or as a narrative and the sum to k+1 expressed in terms of $k + 1$ (or with the expression in term of $k + 1$ stated earlier – it must be seen at some stage). Allow slips in notation if the intent is correct. Depends on all except the B mark, though an attempt at checking the $n = 1$ case must have been made.
(ii) **Way 1**
B1: Shows that f(1) = 15
M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts f(k + 2)

A1: Correctly obtains 16f(k) **or** $45\times5^{k-1}+120\times6^{k-1}$
A1: Reaches a correct expression for f(k + 2) in terms of f(k)
A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow "true for all n" where n represents natural numbers, as this is incorrect.
Way 2
B1: Shows that f(1) = 15
M1: Makes a statement that assumes the result is true for some value of n (Assume (true for) $n = k$ is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts f(k + 2) – f(k) or equivalent work
A1: Achieves a correct expression for f(k + 2) – f(k) in terms of f(k)
A1: Reaches a correct expression for f(k + 2) in terms of f(k)
A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow "true for all n" where n represents natural numbers, as this is incorrect.

Way 3
B1: Shows that f(1) = 15
M1: Makes a statement that assumes the result is true for some odd value of n (Assume (true for) n = 2k + 1 is sufficient (or may use $2k-1$ – note that this may be recovered in their conclusion if they say e.g. if true for $n = k$ then … etc.)
M1: Attempts f(2k+3) (like Way 1), or f(2k +3) – f(2k+1) or equivalent work (like Way 2) It must be the correct f(2k+1) used, in the latter case (subtracting $4^k+5^k+6^k$ instead of $4^{2k+1}+5^{2k+1}+6^{2k+1}$ is M0)
A1: Reaches $16\left(4^{2k+1}+5^{2k+1}+6^{2k+1}\right)+9\times5^{2k+1}+20\times6^{2k+1}$ or suitable equivalent OR achieves a correct expression for f(2k +3) – f(2k+1) in terms of f(2k+1) where factors of 15 are apparent.
A1: Reaches $16\left(4^{2k+1}+5^{2k+1}+6^{2k+1}\right)+45\times5^{2k}+120\times6^{2k}$ or a suitable equivalent OR a correct expression for f(2k + 3) of form $A\mathrm{f}(2k+1)+15(...)$ (though the 15(…) could be separate multiples of 15 where the 15 need not yet be extracted).
A1: Correct conclusion, including explanation that all terms on RHS are divisible by 15, or writing terms each explicitly as multiples of 15. This mark is dependent on all previous marks apart from the B mark. It is gained by conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution. Do not allow "true for all n" where n represents natural numbers, as this is incorrect, but if n represents odd numbers than allow.
Note: $4^{2k}=16^k$ etc may be used throughout.
Note: Way 3 should apply to cases where the question is rephrased in terms of $4^{2n+1}+5^{2n+1}+6^{2n+1}$ for all n (rather than odd n).
Accept use of alternative equivalent language throughout.

Question	Scheme	Marks	AOs
7(a)	$(1+t)\frac{\mathrm{d}P}{\mathrm{d}t}+P=t^{\frac{1}{2}}(1+t)\Rightarrow\frac{\mathrm{d}P}{\mathrm{d}t}+\frac{P}{1+t}=t^{\frac{1}{2}}$	B1	1.1b
	$I=\mathrm{e}^{\int\frac{1}{1+t}\,\mathrm{d}t}=1+t\Rightarrow P(1+t)=\int t^{\frac{1}{2}}(1+t)\,\mathrm{d}t=\ldots$	M1	3.1b
	$P(1+t)=\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+c$	A1	1.1b
	$t=0,\ P=5\Rightarrow c=5$	M1	3.4
	$P=\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}=\dfrac{\frac{2}{3}8^{\frac{3}{2}}+\frac{2}{5}8^{\frac{5}{2}}+5}{9}=\ldots$	M1	1.1b
	= 10 277 bacteria (allow awrt 10 300)	A1	2.2b
		(6)	
(b)	$P=\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}\Rightarrow\frac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{(1+t)\left(t^{\frac{1}{2}}+t^{\frac{3}{2}}\right)-\left(\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5\right)}{(1+t)^2}$ Alt: $P+(1+t)\frac{\mathrm{d}P}{\mathrm{d}t}=t^{\frac{1}{2}}+t^{\frac{3}{2}}\Rightarrow\frac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{t^{\frac{1}{2}}+t^{\frac{3}{2}}-\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}}{(1+t)}$	M1 A1ft	3.4 1.1b
	$\left(\frac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=1}=\frac{\mathrm{d}P}{\mathrm{d}t}=\dfrac{5\times10-\left(\frac{16}{3}+\frac{64}{5}+5\right)}{(5)^2}=\frac{403}{375}$	dM1	3.1a
	$\frac{403}{375}\times1000=\frac{3224}{3}\left(=awrt\,1070\right)$ bacteria per hour	A1	3.2a
		(4)	
	(b) Alternative:		
	$P=\dfrac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}=\dfrac{\frac{16}{3}+\frac{64}{5}+5}{(1+4)}$	M1	3.4
	$=\frac{347}{75}$	A1ft	1.1b
	$(1+t)\frac{\mathrm{d}P}{\mathrm{d}t}+P=t^{\frac{1}{2}}(1+t)\Rightarrow5\frac{\mathrm{d}P}{\mathrm{d}t}+\frac{347}{75}=2\times5\Rightarrow\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{403}{375}$	dM1	3.1a
	$\frac{403}{375}\times1000=\frac{3224}{3}\left(=1075\right)$ bacteria per hour	A1	3.2a
		(4)	
(c)	E.g. • The number of bacteria increases indefinitely which is not realistic	B1	3.5b
		(1)	
			(11 marks)

Notes
(a) B1: A correct rearrangement (may be implied by subsequent work). Alternatively, recognises the LHS as a derivative and writes $(1+t)\frac{dP}{dt}+P=\frac{d}{dt}\left(P(1+t)\right)\left(=t^{\frac{1}{2}}(1+t)\right)$ (may be implied). M1: Uses the model to find the integrating factor (or recognise the derivative) and attempts the solution of the differential equation to achieve $P\times\text{their IF}=\int \text{their}\, t^{\frac{1}{2}}\times\text{their IF}\,dt=...$ but do not be too concerned with the mechanics of integrating the RHS but it must be attempted. A1: Correct solution M1: Interprets the initial conditions to find the constant of integration. Must be using $t = 0$ and $P = 5$ in an equation with a constant of integration, but their equation may have come from incorrect work. This is correctly interpreting the initial conditions and attempting to use them. M1: Uses their solution to the problem to find the population after 8 hours. Must be using their solution, but allow for any equations which arise from an attempt at solving the differential equation. A1: **cso** Correct number of bacteria (accept awrt 10 300) from a correct equation (b) M1: Realises the need to differentiate the model and uses an appropriate method to find the derivative. Allow the M for attempts at implicit differentiation with (1+t)P = ... Trivialised differentiation from incorrect work is M0. A1ft: Correct differentiation of the correct answer to (a) up to the constant of integration to obtain dP/dt in terms of t (if implicit differentiation is used, they must get to a function in terms of t only, or revert to the Alternative method). Follow through on their c in an otherwise correct equation from (a). M1: Uses t = 4 in their dP/dt (allow from any attempts at the derivative) to obtain a value for dP/dt. A1: Correct answer, allow 1075 or answers rounding down to 1070 with correct units. Accept as 1.07 thousand bacteria per hour. (NB If 5000 is used in (a) instead of 5, the answer here would be −198.725) **Alternative:** M1: Substitutes t = 4 into their P A1ft: Correct value for P. Follow through on their constant of integration from part (a), but the rest of the equation must be correct. M1: Uses t = 4 and their P to find a value for dP/dt A1: Correct answer allow 1075 or answers rounding down to 1070 with correct units. Accept as 1.07 thousand bacteria per hour. (c) B1: Suggests a suitable limitation which must refer to the model. Allow for a sensible comment even if they have no equation for the model Do not allow answers such as “the model does not take account of external factors such as temperature” as we do not know what factors the model does take account of.