Mark Scheme (Result)

October 2020

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/3D

October 2020
Publications Code 9FM0_3D_2010_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

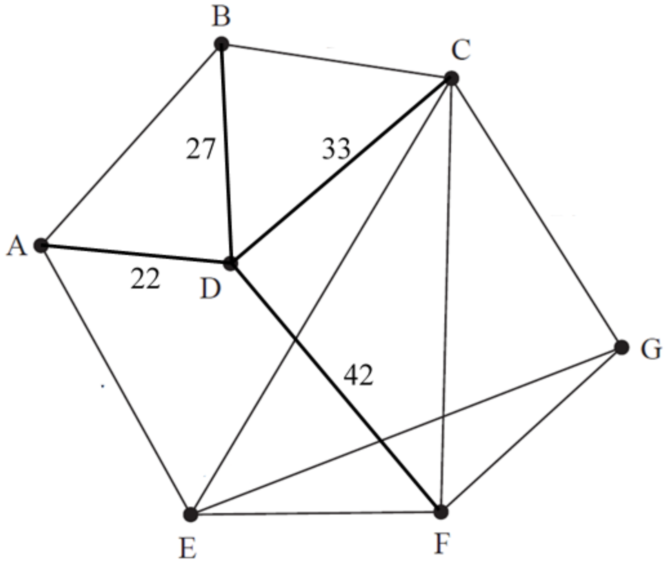
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

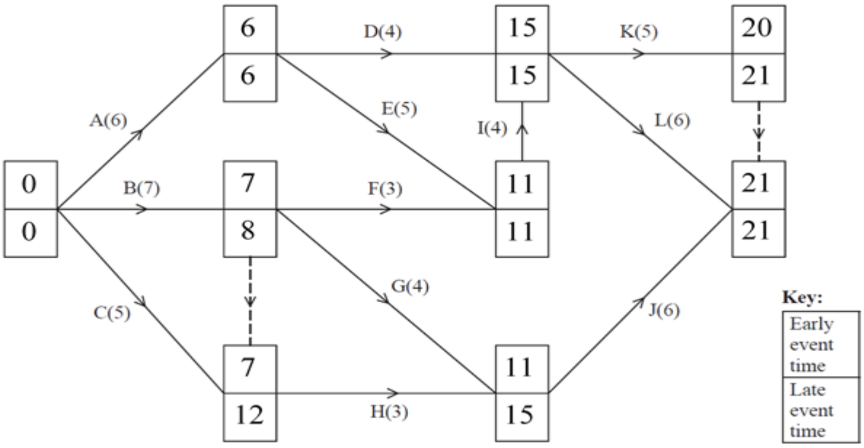
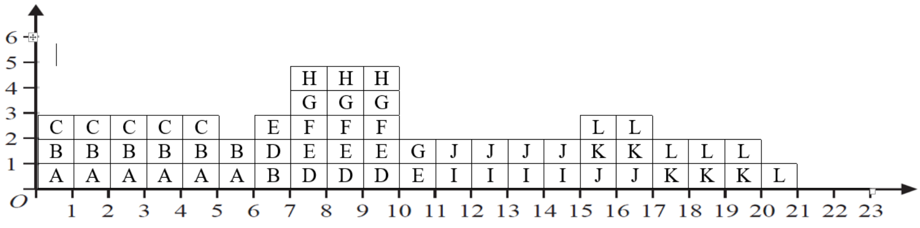
- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- [The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(a)		M1 A1	1.1b 1.1b
		(2)	
(b)	Kruskal's algorithm: AD, AB, BC, CG, reject BD, EG, reject CE, reject CD, reject AE, CF (reject EF, reject FG, reject DF)	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)	Weight of MST: 162 (km)	B1	1.1b
		(1)	
			(6 marks)

Notes for Question 1

(a)
M1: Either all arcs correct (ignore weights) or two arcs correct (including correct weights)
A1: CAO
(b)
M1: Kruskal's algorithm – first three arcs correctly chosen and at least one rejection seen at some point
A1: All six arcs selected correctly AD, AB, BC, CG, EG, CF only
A1: CSO – all selections and rejections correct (in correct order and at the correct time)
(c)
B1: CAO (condone lack of units)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>2(a)</td><td>The dummy at the end of activity B is required as F (and G) are dependent on activity B only, but activity H is dependent on both activities B and C
The dummy at the end of activity K is required as two activities cannot start at the same event and finish at the same event</td><td>B1
B1</td><td>2.4
2.4</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>Activity</th><th>IPA</th><th>Activity</th><th>IPA</th><th>Activity</th><th>IPA</th></tr>
<tr><td>A</td><td>-</td><td>E</td><td>A</td><td>I</td><td>E, F</td></tr>
<tr><td>B</td><td>-</td><td>F</td><td>B</td><td>J</td><td>G, H</td></tr>
<tr><td>C</td><td>-</td><td>G</td><td>B</td><td>K</td><td>D, I</td></tr>
<tr><td>D</td><td>A</td><td>H</td><td>B, C</td><td>L</td><td>D, I</td></tr>
</table>
</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)(i)</td><td>
</td><td>M1
A1
M1
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td>(c)(ii)</td><td>Minimum completion time: 21 hours</td><td>A1ft</td><td>1.1b</td></tr>
<tr><td>(c)(iii)</td><td>Critical activities: A, E, I, L</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(d)</td><td>e.g.

</td><td>M1
A1
A1</td><td>1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
</table>

(e)	Currently five workers are required between time 7 and 10 and so one of the non-critical activities D, F, G or H would have to be delayed and start after time 10 e.g. Activity H could be delayed and start at time 10 (as it has sufficient total float and can finish as late as time 15) and so the project can be completed with fewer workers than the number indicated by the resource histogram as J could be delayed too and start at time 15	M1 A1	2.4 2.2a
		(2)	
(15 marks)			
Notes for Question 2			

(a)

B1: Correct explanation for precedence dummy (must mention B, C, H and one of F or G)

B1: Correct explanation for uniqueness dummy

(b)

B1: Six correct rows (not including A, B and C)

B1: All rows correct (accept blanks for A, B and C)

(c)(i)

M1: All top boxes completed, number generally increasing L to R (condone one “rogue”)

A1: CAO - Top boxes

M1: All bottom boxes completed, numbers generally decreasing R to L (condone one “rogue”) – condone lack of 0 or 21 for the M mark only

A1: CAO - Bottom boxes

(c)(ii)

A1ft: Correct follow through from their completed top boxes

(c)(iii)

A1: Correct critical activities (A, E, I and L only)

(d)

M1: Plausible histogram (correct up to time 6) with no holes or overhangs (must go to at least 20 on the time axis)

A1: Histogram correct to time 10

A1: Histogram correct from time 10 to time 21

(e)

M1: Explanation involving the need to delay just one of the non-critical activities (must mention one of D, F, G or H) to start after time 10 (oe) – follow through their histogram

A1: Dependent on a correct histogram and correct answer to (c)(i). Correct deduction that it is possible to complete with fewer workers e.g. delay H to start at 10 therefore delay J to start at its late time (or 15) – A0 if mention of delaying activity F

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(a)</td><td>
Time matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>8</td><td>4</td><td>7</td><td>∞</td></tr>
<tr><td>B</td><td>8</td><td>-</td><td>3</td><td>∞</td><td>10</td></tr>
<tr><td>C</td><td>4</td><td>3</td><td>-</td><td>∞</td><td>6</td></tr>
<tr><td>D</td><td>7</td><td>∞</td><td>1</td><td>-</td><td>1</td></tr>
<tr><td>E</td><td>∞</td><td>10</td><td>6</td><td>1</td><td>-</td></tr>
</table>
Route matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table>
</td><td>B1
B1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>
Time matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>7</td><td>4</td><td>7</td><td>10</td></tr>
<tr><td>B</td><td>7</td><td>-</td><td>3</td><td>14</td><td>9</td></tr>
<tr><td>C</td><td>4</td><td>3</td><td>-</td><td>11</td><td>6</td></tr>
<tr><td>D</td><td>5</td><td>4</td><td>1</td><td>-</td><td>1</td></tr>
<tr><td>E</td><td>10</td><td>9</td><td>6</td><td>1</td><td>-</td></tr>
</table>
Route matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>C</td><td>C</td><td>D</td><td>C</td></tr>
<tr><td>B</td><td>C</td><td>B</td><td>C</td><td>C</td><td>C</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>A</td><td>E</td></tr>
<tr><td>D</td><td>C</td><td>C</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>C</td><td>C</td><td>C</td><td>D</td><td>E</td></tr>
</table>
Time matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>7</td><td>4</td><td>7</td><td>8</td></tr>
<tr><td>B</td><td>7</td><td>-</td><td>3</td><td>14</td><td>9</td></tr>
<tr><td>C</td><td>4</td><td>3</td><td>-</td><td>11</td><td>6</td></tr>
<tr><td>D</td><td>5</td><td>4</td><td>1</td><td>-</td><td>1</td></tr>
<tr><td>E</td><td>6</td><td>5</td><td>2</td><td>1</td><td>-</td></tr>
</table>
Route matrix
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>C</td><td>C</td><td>D</td><td>D</td></tr>
<tr><td>B</td><td>C</td><td>B</td><td>C</td><td>C</td><td>C</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>A</td><td>E</td></tr>
<tr><td>D</td><td>C</td><td>C</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>D</td><td>D</td><td>D</td><td>D</td><td>E</td></tr>
</table>
</td><td>M1
A1
M1
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(c)(i)
(ii)
(iii)</td><td>NNA: A – C – B – E – D – A
4 + 3 + 9 + 1 + 5 = 22 minutes
A – C – B – C – E – D – C – A</td><td>B1
dB1
B1</td><td>1.1b
1.1b
3.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(9 marks)</td></tr>
<tr><td colspan="4" align="center">Notes for Question 3</td></tr>
<tr><td colspan="4">(a)
B1: Correct time matrix
B1: Correct route matrix</td></tr>
</table>

(b)

M1: No change in the third row and third column of both matrices with at least one value in the time matrix reduced correctly and one value in the route matrix changed to C

A1: CAO

M1: No change in the fourth row and fourth column of both matrices with at least one value in the time matrix reduced correctly (follow through their first iteration) and one value in the route matrix changed to D

A1: CAO

(c)(i)

B1: CAO

(c)(ii)

dB1: CAO – not from A – C – D – E – B – A

(c)(iii)

B1: CAO

Question	Scheme	Marks	AOs
4(a)	$2y \le 5x,\ y \ge x+1,\ 6x+5y \le 30$	B2,1,0	1.1b 2.5
		(2)	
(b)	$\left(\frac{2}{3},\frac{5}{3}\right),\left(\frac{60}{37},\frac{150}{37}\right),\left(\frac{25}{11},\frac{36}{11}\right)$ $\left(\frac{2}{3},\frac{5}{3}\right) \rightarrow P=\frac{11}{3}$ $\left(\frac{60}{37},\frac{150}{37}\right) \rightarrow P=\frac{330}{37}$ $\left(\frac{25}{11},\frac{36}{11}\right) \rightarrow P=\frac{111}{11}$ so optimal vertex is $\left(\frac{25}{11},\frac{36}{11}\right)$	B1 B1 M1 A1	1.1b 1.1b 2.1 2.2a
		(4)	
(c)	$Q=3x+ay$ $3\left(\frac{25}{11}\right)+\frac{36a}{11}>3\left(\frac{60}{37}\right)+\frac{150a}{37}$ $\Rightarrow a<\frac{5}{2}$ $3\left(\frac{25}{11}\right)+\frac{36a}{11}>3\left(\frac{2}{3}\right)+\frac{5a}{3}$ $\Rightarrow a>-3$	 M1 A1 M1 A1	 3.1a 2.2a 1.1b 2.2a
		(4)	
			(10 marks)

Notes for Question 4

(a)

B1: Any two correct (accept strict inequalities) – accept equivalent inequalities

B1: CAO (accept equivalent inequalities but inequalities must not be strict)

(b)

B1: One correct vertex (must be exact)

B1: All three correct vertices (must be exact)

M1: Testing all three of their vertices in the correct objective function

A1: Correct three values of *P* and correct optimal vertex either stated or clearly indicated on the graph

(c)

M1: Their optimal point from (b) evaluated in Q compared to their $\left(\frac{60}{37}, \frac{150}{37}\right)$ evaluated in Q (with correct inequality)

A1: $a < \frac{5}{2}$

M1: Their optimal point from (b) evaluated in Q compared to their $\left(\frac{2}{3}, \frac{5}{3}\right)$ evaluated in Q (with correct inequality)

A1: $a > -3$

Question	Scheme	Marks	AOs
5(a)	If *x* has been placed in Bin 2 then $10 < x \le 31$ - this is because Bin 1 at this stage only contains 40 and before *x* had been placed in Bin 2 it only contained 19		
	As the 18 has been placed in Bin 3 this implies that $x > 50 - (19 + 18)$ so $x > 13$	B1	3.1a
	As the 10 has been placed in Bin 2 after the *x* then $x \le 50 - (19 + 10)$ so $x \le 21$	B1	2.4
	However, the number are all distinct and therefore $13 < x < 21$	B1	2.2a
		(3)	
(b)	$13 < x < 17$	B1 B1	2.2a 2.2a
		(2)	
(c)	If *x* has been placed in Bin 3 then this implies that $x \le 15$	M1	2.4
	So *x* is either 14 or 15 – but as Bin 2 is full $\Rightarrow x = 14$	A1	2.2a
		(2)	
			(7 marks)

Notes for Question 5

(a)

B1: Correct reasoning of why $x > 13$ accept $x > 50 - (19 + 18)$

B1: Correct explanation of why $x \le 21$ accept $x \le 50 - (19 + 10)$

B1: Correct deduction that $13 < x < 21$ must mention that the numbers are distinct (oe)

(b)

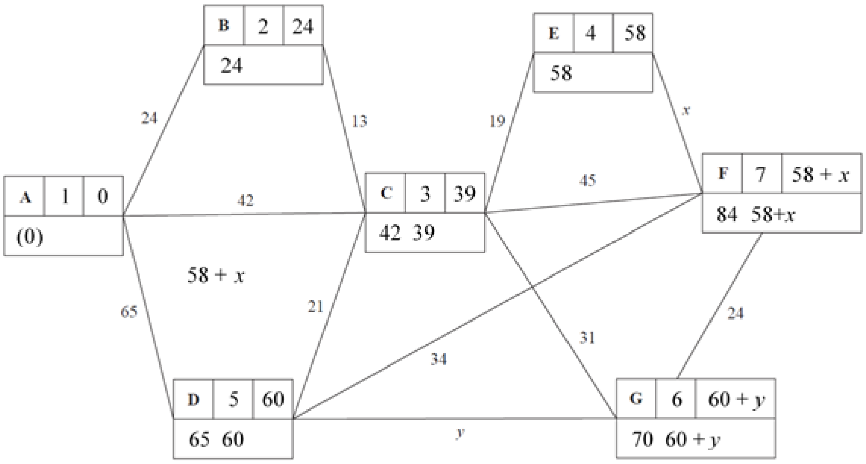
B1: Use first complete pass to deduce that $x < 17$

B1: Correct lower bound of $x > 13$

(c)

M1: Using first-fit decreasing in an attempt to derive new upper bound for *x* (so either for stating both *x* could equal 14 or 15, $x \le 15$ or $x < 15$)

A1: Correct deduction that $x = 14$ (must clearly state or imply that Bin 2 is full)

Question	Scheme	Marks	AOs
6(a)	The graph has exactly two odd nodes and so the graph is semi-Eulerian	B1 dB1	2.4 2.2a
		(2)	
(b)		M1 A1 A1 A1	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to F is 58 + x and shortest path from A to G is 60 + y	A1ft	2.2a
	58 + x + 60 + y = 140	M1	2.1
	The only odd nodes in the network are A and G	B1	2.2a
	Route inspection algorithm: Shortest route between A and G is 60 + y $\Rightarrow$ 320 + x + y + 60 + y = 409	M1	3.1b
	x = 15 and y = 7	A1	2.2a
		(9)	
			(11 marks)

Notes for Question 6

(a)

B1: Explanation which consists of the graph having two odd nodes or stating graph is semi-Eulerian

dB1: Exactly two odd nodes (or two odd nodes and five even nodes or the rest even) together with the deduction that therefore the graph is semi-Eulerian

(b)

M1: For a larger number replaced by a smaller one in two working value boxes at C, D, G or F

A1: For all values correct (and in correct order) at A, B and C

A1: For all values correct (and in correct order) at E and D

A1: For all values correct (and in correct order) at G and F

A1ft: Length of shortest path from A to F or A to G stated (may be seen in an equation(s))

M1: (length of shortest path from A to F) + (length of shortest path from A to G) = 140 – linear equation in x and y

B1: Correctly stating the two odd nodes (A and G) – could be implied by subsequent working

M1: For an equation based on the route from A to G (320 + x + y + final value at G (in y) = 409)

A1: CAO for x and y

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7(a)</td><td>Maximise $(P=)2x+y+3z$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$x+2y+3z\le 45$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$3x+2y\ge 9$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td>$-x+4z\ge 4$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(b)</td><td>$A=-(a_1+a_2)\Rightarrow -(9-3x-2y+s_2+4+x-4z+s_3)$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$A-2x-2y-4z+s_2+s_3=-13$ therefore bottom row of the table is
<table><tr><td>A</td><td>–2</td><td>–2</td><td>–4</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>–13</td></tr></table></td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)(i)</td><td>In the given tableau the value of the objective <i>A</i> is equal to zero indicating that a basic feasible solution has been found</td><td>B1</td><td>2.4</td></tr>
<tr><td>(c)(ii)</td><td>$x=3, y=0, z=\frac{7}{4}, s_1=\frac{147}{4}, s_2=s_3=0$</td><td>B1
B1</td><td>3.4
1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>
<table>
<tr><th>b.v</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>Value</th><th>Row Ops</th></tr>
<tr><td>s_2</td><td>0</td><td>$\frac{10}{7}$</td><td>0</td><td>$\frac{12}{7}$</td><td>1</td><td>$\frac{9}{7}$</td><td>63</td><td>R1 $\div\frac{7}{12}$</td></tr>
<tr><td>x</td><td>1</td><td>$\frac{8}{7}$</td><td>0</td><td>$\frac{4}{7}$</td><td>0</td><td>$\frac{3}{7}$</td><td>24</td><td>R2 + $\frac{1}{3}$R1</td></tr>
<tr><td>z</td><td>0</td><td>$\frac{2}{7}$</td><td>1</td><td>$\frac{1}{7}$</td><td>0</td><td>$-\frac{1}{7}$</td><td>7</td><td>R3 + $\frac{1}{12}$R1</td></tr>
<tr><td>P</td><td>0</td><td>$\frac{15}{7}$</td><td>0</td><td>$\frac{11}{7}$</td><td>0</td><td>$\frac{3}{7}$</td><td>69</td><td>R4 + $\frac{11}{12}$R1</td></tr>
</table></td><td>M1
A1
M1
A1
A1</td><td>2.1
1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(e)(i)</td><td>Yes, an optimal solution has been found as there are no negative values in the objective (<i>P</i>) row</td><td>B1</td><td>2.4</td></tr>
<tr><td>(e)(ii)</td><td><i>P</i> = 69</td><td>B1ft</td><td>3.4</td></tr>
<tr><td>(e)(iii)</td><td>$s_2=63, x=24, z=7$</td><td>B1ft</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4">(17 marks)</td></tr>
</table>

Notes for Question 7

(a)

B1: CAO – including maximise (or max)

B1: CAO (oe)

B1: CAO (oe)

B1: CAO (oe)

(b)

M1: Setting up the new objective and substituting for a_1 and a_2

A1: Correct values substituted into Table 1

(c)

B1: CAO – mention that $A = 0$

B1: At least three values stated correctly

B1: All six values correct (ignore values stated for a_1, a_2 and P)

(d)

M1: Correct pivot located, attempt to divide row

A1: Pivot row correct including change of b.v.

M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (y, s_1, s_3 or value) correct following through their choice of pivot from column s_2 or s_3

A1: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (y, s_1, s_3 or value) correct

A1: CAO all values and row operations correctly stated – allow if row operations given in terms of old row 1 – **ignore b.v. column for this mark**

(e)(i)

B1: Correct reasoning of why solution is optimal or using $P = 69 - \frac{15}{7}y - \frac{11}{7}s_1 - \frac{3}{7}s_3$ and mentioning increasing y, s_1, s_3 would decrease P (oe)

(e)(ii)

B1ft: their value of P – dependent on both M marks in (d)

(e)(iii)

B1ft: their values of the basic variables **only** – dependent on both M marks in (d)