

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel  
Level 3 GCE**

Centre Number

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Candidate Number

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Time 1 hour 30 minutes

Paper  
reference

**9FM0/4C**



# **Further Mathematics**

**Advanced**

## **PAPER 4C: Further Mechanics 2**

**You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.  
Calculators must not have the facility for symbolic algebra manipulation,  
differentiation and integration, or have retrievable mathematical formulae  
stored in them.**

### **Instructions**

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$  and give your answer to either 2 significant figures or 3 significant figures.

### **Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

### **Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

**Turn over ▶**

**P66805A**

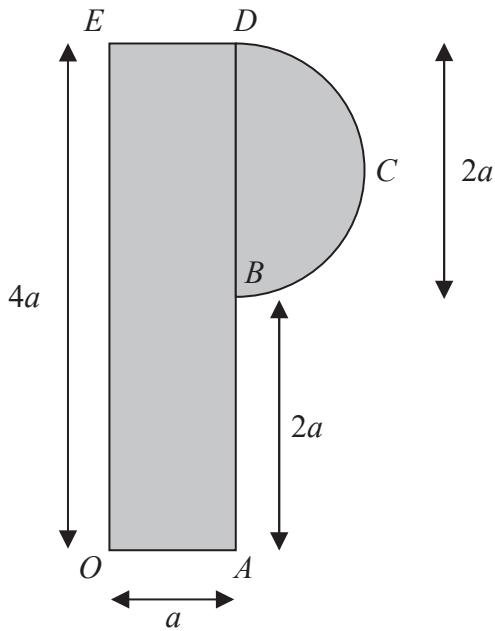
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1/1/1/1/1/



**Pearson**

1.

**Figure 1**

A letter P from a shop sign is modelled as a uniform plane lamina which consists of a rectangular lamina,  $OABDE$ , joined to a semicircular lamina,  $BCD$ , along its diameter  $BD$ .

$OA = ED = a$ ,  $AB = 2a$ ,  $OE = 4a$ , and the diameter  $BD = 2a$ , as shown in Figure 1.

Using the model,

(a) find, in terms of  $\pi$  and  $a$ , the distance of the centre of mass of the letter P,

from (i)  $OE$

(ii)  $OA$

(6)

The letter P is freely suspended from  $O$  and hangs in equilibrium. The angle between  $OE$  and the downward vertical is  $\alpha$ .

Using the model,

(b) find the exact value of  $\tan \alpha$

(2)



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**Question 1 continued**

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**(Total for Question 1 is 8 marks)**



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- DO NOT WRITE IN THIS AREA
2. At time  $t = 0$ , a small stone  $P$  of mass  $m$  is released from rest and falls vertically through the air. At time  $t$ , the speed of  $P$  is  $v$  and the resistance to the motion of  $P$  from the air is modelled as a force of magnitude  $kv^2$ , where  $k$  is a constant.

(a) Show that  $t = \frac{V}{2g} \ln\left(\frac{V+v}{V-v}\right)$  where  $V^2 = \frac{mg}{k}$  (4)

(b) Give an interpretation of the value of  $V$ , justifying your answer. (2)

At time  $t$ ,  $P$  has fallen a distance  $s$ .

(c) Show that  $s = \frac{V^2}{2g} \ln\left(\frac{V^2}{V^2 - v^2}\right)$  (4)

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**Question 2 continued**

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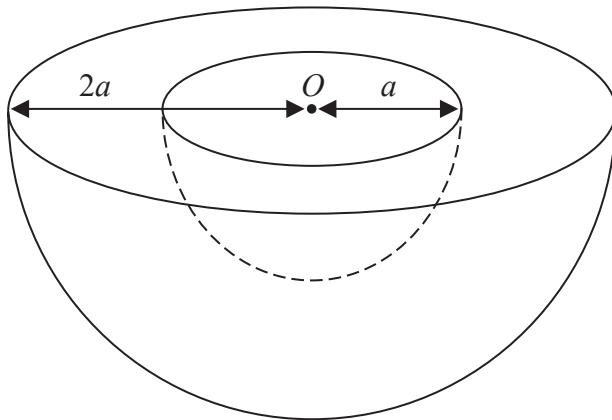
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**(Total for Question 2 is 10 marks)**



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3.

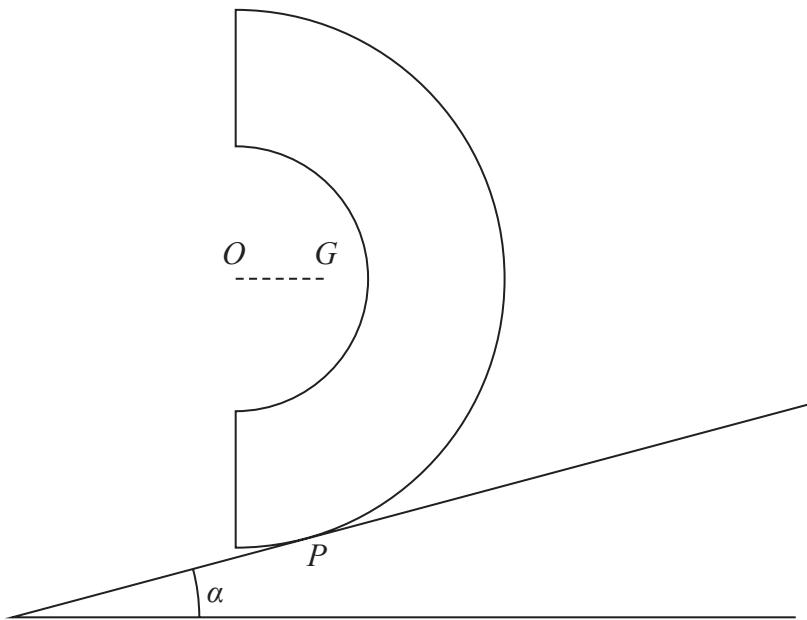
**Figure 2**

A uniform solid hemisphere  $H$  has radius  $2a$ . A solid hemisphere of radius  $a$  is removed from the hemisphere  $H$  to form a bowl. The plane faces of the hemispheres coincide and the centres of the two hemispheres coincide at the point  $O$ , as shown in Figure 2.

The centre of mass of the bowl is at the point  $G$ .

- (a) Show that  $OG = \frac{45a}{56}$  (4)

Figure 3 below shows a cross-section of the bowl which is resting in equilibrium with a point  $P$  on its curved surface in contact with a rough plane. The plane is inclined to the horizontal at an angle  $\alpha$  and is sufficiently rough to prevent the bowl from slipping. The line  $OG$  is horizontal and the points  $O$ ,  $G$  and  $P$  lie in a vertical plane which passes through a line of greatest slope of the inclined plane.

**Figure 3**

- (b) Find the size of  $\alpha$ , giving your answer in degrees to 3 significant figures.

(2)

**Question 3 continued**

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P 6 6 8 0 5 A 0 1 1 3 2

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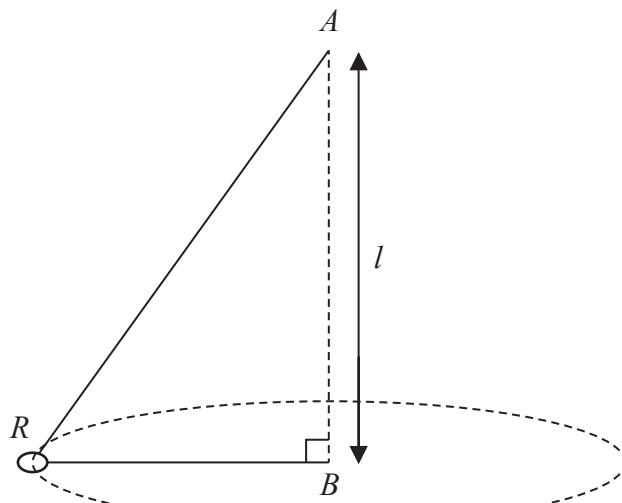


Figure 4

One end of a light inextensible string of length  $2l$  is attached to a fixed point  $A$ . A small smooth ring  $R$  of mass  $m$  is threaded on the string and the other end of the string is attached to a fixed point  $B$ . The point  $B$  is vertically below  $A$ , with  $AB = l$ . The ring is then made to move with constant speed  $V$  in a horizontal circle with centre  $B$ . The string is taut and  $BR$  is horizontal, as shown in Figure 4.

- (a) Show that  $BR = \frac{3l}{4}$  (2)

Given that air resistance is negligible,

- (b) find, in terms of  $m$  and  $g$ , the tension in the string, (4)
- (c) find  $V$  in terms of  $g$  and  $l$ . (4)



**Question 4 continued**

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**Question 4 continued**

Handwriting practice lines for Question 4.

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**Question 4 continued**

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**(Total for Question 4 is 10 marks)**



P 6 6 8 0 5 A 0 1 7 3 2

5. A light inextensible string of length  $a$  has one end attached to a fixed point  $O$ . The other end of the string is attached to a small stone of mass  $m$ . The stone is held with the string taut and horizontal. The stone is then projected vertically upwards with speed  $U$ .

The stone is modelled as a particle and air resistance is modelled as being negligible.

Assuming that the string does not break, use the model to

- (a) find the least value of  $U$  so that the stone will move in complete vertical circles.

(6)

The string will break if the tension in it is equal to  $\frac{11mg}{2}$

Given that  $U = 2\sqrt{ag}$ , use the model to

- (b) find the total angle that the string has turned through, from when the stone is projected vertically upwards, to when the string breaks,

(6)

- (c) find the magnitude of the acceleration of the stone at the instant just before the string breaks.

(4)

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**(Total for Question 5 is 16 marks)**



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6. A light elastic string, of natural length  $l$  and modulus of elasticity  $2mg$ , has one end attached to a fixed point  $A$  and the other end attached to a particle  $P$  of mass  $m$ . The particle  $P$  hangs in equilibrium at the point  $O$ .

(a) Show that  $AO = \frac{3l}{2}$  (2)

The particle  $P$  is pulled down vertically from  $O$  to the point  $B$ , where  $OB = l$ , and released from rest.

Air resistance is modelled as being negligible.

Using the model,

- (b) prove that  $P$  begins to move with simple harmonic motion about  $O$  with

period  $\pi \sqrt{\frac{2l}{g}}$  (5)

The particle  $P$  first comes to instantaneous rest at the point  $C$ .

Using the model,

- (c) find the length  $BC$  in terms of  $l$ , (4)

- (d) find, in terms of  $l$  and  $g$ , the exact time it takes  $P$  to move directly from  $B$  to  $C$ . (5)



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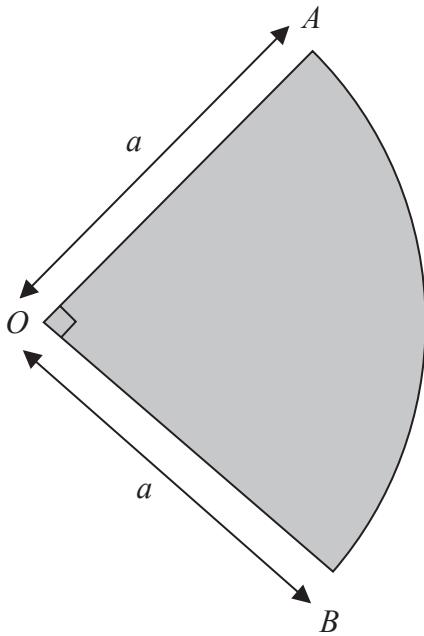
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**(Total for Question 6 is 16 marks)**



7. [In this question, you may assume that the centre of mass of a circular arc, radius  $r$ , with angle at centre  $2\alpha$ , is a distance  $\frac{r \sin \alpha}{\alpha}$  from the centre.]



**Figure 5**

A thin non-uniform metal plate is in the shape of a sector  $OAB$  of a circle with centre  $O$  and radius  $a$ . The angle  $AOB = \frac{\pi}{2}$ , as shown in Figure 5.

The plate is modelled as a non-uniform lamina.

The mass per unit area of the lamina, at any point  $P$  of the lamina, is modelled as

$$k(OP)^2, \text{ where } k = \frac{4\lambda}{\pi a^4} \text{ and } \lambda \text{ is a constant.}$$

Using the model,

- (a) find the mass of the plate in terms of  $\lambda$ , (5)  
(b) find, in terms of  $a$ , the distance of the centre of mass of the plate from  $O$ . (4)

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