



Oxford Cambridge and RSA

Thursday 16 May 2019 – Afternoon

AS Level Further Mathematics A

Y534/01 Discrete Mathematics

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae AS Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

- 1 Alfie has a set of 15 cards numbered consecutively from 1 to 15. He chooses two of the cards.

(a) How many different sets of two cards are possible? [2]

Alfie places the two cards side by side to form a number with 2, 3 or 4 digits.

(b) Explain why there are fewer than ${}^{15}P_2 = 210$ possible numbers that can be made. [1]

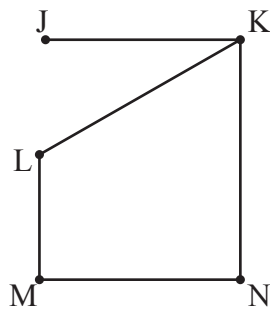
(c) Explain why, with these cards, 1 is the lead digit more often than any other digit. [1]

Alfie makes the number 113, which is a 3-digit prime number.

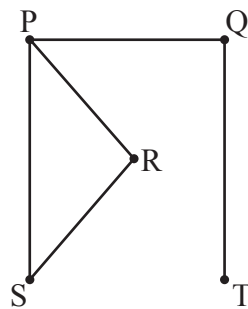
Alfie says that the problem of working out how many 3-digit prime numbers can be made using two of the cards is a construction problem, because he is trying to find all of them.

(d) Explain why Alfie is wrong to say this is a construction problem. [1]

- 2 Two graphs are shown below.



Graph G1



Graph G2

(a) List the vertex degrees for each graph. [3]

(b) Prove that the graphs are non-isomorphic. [2]

The two graphs are joined together by adding an arc connecting J and T.

(c) (i) Explain how you know that the resulting graph is not Eulerian. [1]

(ii) Describe how the graph can be made Eulerian by adding one more arc. [1]

The vertices of the graph K_3 are connected to the vertices of the graph K_4 to form the graph K_7 .

(d) Explain why 12 arcs are needed connecting K_3 to K_4 . [3]

- 3 (a) Give an example of a standard sorting algorithm that can be used when some of the values are not known until after the sorting has been started. [1]

Becky needs to sort a list of numbers into increasing order.
She uses the following algorithm:

- STEP 1: Let L be the first value in the input list.
Write this as the first value in the output list and delete it from the input list.
- STEP 2: If the input list is empty go to STEP 7.
Otherwise let N be the new first value in the input list and delete this value from the input list.
- STEP 3: Compare N with L .
- STEP 4: If N is less than or equal to L
- write the value of N immediately before L in the output list,
 - replace L with the first value in the new output list,
 - then go to STEP 2.
- STEP 5: If N is greater than L
- if L is the value of the last number in the output list, go to STEP 6;
 - otherwise, replace L with the next value in the output list and then go to STEP 3.
- STEP 6: Write the value of N immediately after L in the output list.
Let L be the first value in the new output list and then go to STEP 2.
- STEP 7: Print the output list and STOP.

- (b) Trace through Becky's algorithm when the input list is

6 9 5 7 6 4

Complete the table in the Printed Answer Booklet, starting a new row each time that STEP 3 or STEP 7 is used.

You should not need all the lines in the Answer Booklet. [5]

Becky measures the efficiency of her sort by counting using the number of times that STEP 3 is used.

- (c) (i) How many times did Becky use STEP 3 in sorting the list from part (b)? [1]
- (ii) What is the greatest number of times that STEP 3 could be used in sorting a list of 6 values? [1]

A computer takes 15 seconds to sort a list of 60 numbers using Becky's algorithm.

- (d) Approximately how long would you expect it to take the computer to sort a list of 300 numbers using the algorithm? [3]

- 4 The table shows the activities involved in a project, their durations in hours and their immediate predecessors. The activities can be represented as an activity network.

Activity	A	B	C	D	E	F	G	H
Duration	2	4	5	4	3	3	2	4
Immediate predecessors	-	A	-	A, C	B, C	B, D	D, E	F, G

- (a) Use standard algorithms to find the activities that form
- the longest path(s)
 - the shortest path(s)

through the activity network.

You must show working to demonstrate the use of the algorithms. [8]

Only one of the paths from part (a) has a practical interpretation.

- (b) What is the practical interpretation of the total weight of that path? [1]

The duration of activity E can be changed. No other durations change.

- (c) What is the smallest increase to the duration of E that will make activity E become part of a longest path through the network? [1]

- 5 Corey is training for a race that starts in 18 hours time. He splits his training between gym work, running and swimming.

- At most 8 hours can be spent on gym work.
- At least 4 hours must be spent running.
- The total time spent on gym work and swimming must not exceed the time spent running.

Corey thinks that time spent on gym work is worth 3 times the same time spent running or 2 times the same time spent swimming. Corey wants to maximise the worth of the training using this model.

- (a) Formulate a linear programming problem to represent Corey's problem. Your formulation must include defining the variables that you are using. [5]

Suppose that Corey spends the maximum of 8 hours on gym work.

- (b) (i) Use a graphical method to determine how long Corey should spend running and how long he should spend swimming. [5]
- (ii) Describe why this solution is not practical. [1]
- (iii) Describe how Corey could refine the LP model to make the solution more realistic. [1]

- 6 Drew and Emma play a game in which they each choose a strategy and then use the tables below to determine the pay-off that each receives.

Drew's pay-off		Emma		
		X	Y	Z
Drew	P	3	14	11
	Q	12	4	7
	R	11	4	6

Emma's pay-off		Emma		
		X	Y	Z
Drew	P	13	2	5
	Q	4	12	9
	R	5	12	10

- (a) Convert the game into a zero-sum game, giving the pay-off matrix for Drew. [2]
- (b) Determine the optimal mixed strategy for Drew. [5]
- (c) Determine the optimal mixed strategy for Emma. [5]

END OF QUESTION PAPER

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