Qualification Accredited

Oxford Cambridge and RSA

A LEVEL

Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y540/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

Paper Y540/01 series overview

This paper, along with Y541, assesses the compulsory core content of the A Level Further Mathematics A – H245 qualification. Questions in each paper can assess any part of the core specification. Due to Covid-19 this is only the second year in which these papers have been sat by a full cohort. There was some evidence that many candidates, while confident with standard techniques, had perhaps not been sufficiently exposed to minor points from the specification and lacked confidence with providing sufficient mathematical justification for the working that was done.

The level of demand of the questions on this paper was much the same as in previous series with some easy questions and some that offered more challenge. There were a number of very good scripts but there were also some fundamental errors seen.

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• were able to answer questions on all parts of the syllabus being tested • showed a secure grasp of all standard techniques • communicated well through work being set out in a logical and ordered way • applied the breadth of their mathematical knowledge to find ways to tackle problems unfamiliar to them.	• showed a limited knowledge of standard techniques across the breadth of the specification • often displayed confusion when progressing through a multi-step question • produced unclear or incomplete mathematical arguments • struggled to bring together different aspects of the course to find solution routes for problems unfamiliar to them.

Question 1 (a)

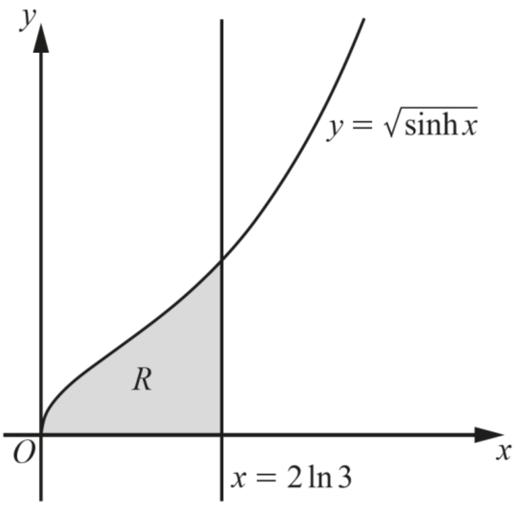
1 In this question you must show detailed reasoning.

(a) Show that $\cosh(2\ln 3) = \frac{41}{9}$. **[2]**

There were no problems with this opening question.

Question 1 (b)

The region R is bounded by the curve with equation $y = \sqrt{\sinh x}$, the x-axis and the line with equation $x = 2\ln 3$ (see diagram). The units of the axes are centimetres.



A manufacturer produces bell-shaped chocolate pieces. Each piece is modelled as being the shape of the solid formed by rotating R completely about the x-axis.

(b) Determine, according to the model, the exact volume of one chocolate piece. **[4]**

The vast majority of candidates scored full marks for this question.

The main errors were (i) a failure to use the correct formula for the volume of the solid of revolution and (ii) substituting 0 for cosh 0.

Question 2 (a), (b), (c), (d)

2 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$.

(a) Calculate det **A**. **[1]**

(b) Write down $\mathbf{A}^{-1}$. **[1]**

(c) Hence solve the equation $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. **[2]**

(d) Write down the matrix **B** such that $\mathbf{AB} = 4\mathbf{I}$. **[1]**

There were no problems with parts (a) to (d) of Question 2.

Question 2 (e)

Matrices **C** and **D** are given by $\mathbf{C} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 0 & 2 & p \end{pmatrix}$ where p is a constant.

(e) Find, in terms of p,

- the matrix **CD**
- the matrix **DC**. **[3]**

A number of candidates seemed not to understand the rule of multiplication that the multiplication of (3x1) x (1x3) gave a (3x3) matrix while the multiplication of (1x3) x (3x1) gave a (1x1) matrix. Some gave a (1x3) and a (3x1) as their answer, while others decided that one of them was not possible.

For those who got the right size for the two matrices, the main error was to fail to express $\mathbf{DC} = (p)$ as a matrix.

Question 2 (f)

It is observed that $\mathbf{CD} \neq \mathbf{DC}$.

(f) The result that $\mathbf{CD} \neq \mathbf{DC}$ is a counter example to the claim that matrix multiplication has a particular property. Name this property. **[1]**

It is a counter example to the claim that matrix multiplication is commutative, thus showing that in general matrix multiplication is not commutative. There were various ways of expressing this which were given credit including some rather poor spelling of "commutative".

Question 3 (a)

3 In this question you must show detailed reasoning.

(a) Find the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

This is a "**detailed reasoning**" question. Whilst this statement does not prohibit the use of a calculator, candidates should understand that what is being tested is mathematical reasoning and not the efficient use of a calculator. In this question a number of candidates wrote their answer with no working at all and so could not be given the marks.

Question 3 (b) (i)

The loci C_1 and C_2 are given by $|z| = |z - 2\mathrm{i}|$ and $|z - 2| = \sqrt{5}$ respectively.

(b) (i) Sketch on a single Argand diagram the loci C_1 and C_2, showing any intercepts with the imaginary axis. **[3]**

Allowance was made for candidates who were drawing freehand – a number of closed regions were accepted as circles even though it was quite a stretch of the imagination. However, there were some parts of the response for which there was no allowance. The circle had to have its centre marked and the line had to be horizontal and pass through the imaginary axis at (0,i), which was the same point as the circle.

Assessment for learning

Section 2b of the H245 specification includes details of the requirements expected for defined command words such as 'sketch'.

An A4 summary can also be found on the qualification page:

https://www.ocr.org.uk/Images/533967-a-level-maths-command-words-poster-a4-size.pdf

Question 3 (b) (ii)

(ii) Indicate, by shading on your Argand diagram, the region

$\{z: |z| \leqslant |z-2\mathrm{i}|\} \cap \{z: |z-2| \leqslant \sqrt{5}\}$. **[1]**

Even those who placed the line or the circle (or both) in the wrong place shaded the region that met the criteria for the diagram.

Question 3 (c) (i)

(c) **(i)** Show that both of the roots of the equation $2z^2 - 2z + 5 = 0$ satisfy $|z-2| < \sqrt{5}$. **[2]**

The "**In this question you must show detailed reasoning**" is stated at the beginning of the question and so applies throughout the whole question. As a "**detailed reasoning**" question the working of the modulus had to be shown as well as a statement of inequality of distance.

Question 3 (c) (ii)

(ii) State, with a reason, which root of the equation $2z^2 - 2z + 5 = 0$ satisfies $|z| < |z-2\mathrm{i}|$. **[1]**

Some candidates stated the correct root but did not provide sufficient reasoning to be awarded the mark.

Question 3 (d)

(d) On the same Argand diagram as part **(b)**, indicate the positions of the roots of the equation $2z^2 - 2z + 5 = 0$. **[2]**

From all that has been asked in the earlier questions, the two points had to be (i) a conjugate pair (ii) both inside the circle and (iii) one above and the other below the axis.

Question 4

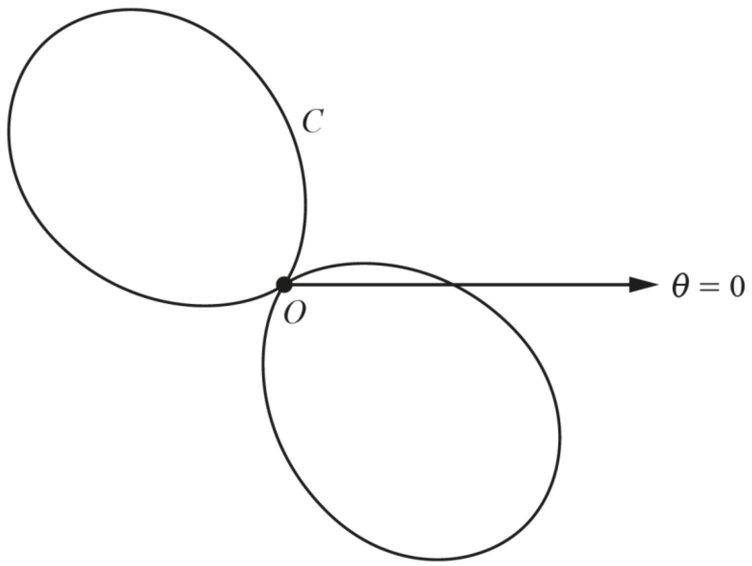
4 Determine the acute angle between the line $\mathbf{r} = \begin{pmatrix} -\sqrt{3} \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2\sqrt{3} \\ -\sqrt{3} \end{pmatrix}$ and the y-axis. **[4]**

It was necessary first to give the direction vector for the y-axis and to identify the direction vector of the line. A number of candidates did not do either one or both of these. Some attempted to use either the position vector of the point that defined the line or some other vector. Those candidates that completed this initial stage and got these two vectors correctly generally found that the result followed by an easy substitution onto the appropriate formula.

A number of candidates did not use the easiest form of the direction vector but a multiple of it and this made the calculation a little more complicated.

Question 5 (a)

5 The diagram below shows the curve C with polar equation $r = 3(1 - \sin 2\theta)$ for $0 \leqslant \theta \leqslant 2\pi$.



(a) Show that a cartesian equation of C is $(x^2 + y^2)^3 = 9(x - y)^4$. **[3]**

Most candidates were able to obtain an equation that did not contain θ but the last step, to eliminate r as well proved too much for some. The presentation of the algebraic manipulation was not always set out in a clear mathematical argument, and often very long winded approaches were taken. Some candidates also found it difficult to cope with the "3" in the equation.

Question 5 (b)

(b) Show that the line with equation $y = x$ is a line of symmetry of C. **[2]**

This question proved quite challenging for the majority of candidates. The easiest way was to swap x and y to show that the same equation resulted, thus demonstrating that if (x, y) satisfied the equation then so did (y, x). The alternative method was to show, using the polar equation that for any point (r, θ) the mirror image in the line $y = x$, which is the point $\left(r, \frac{\pi}{4} - \theta\right)$ also satisfied it. Many candidates attempted to find tangents at the origin which did not prove this property.

Question 5 (c)

(c) **In this question you must show detailed reasoning.**

Find the exact area of each of the loops of C. **[6]**

Most candidates used the correct formula for finding area and managed to deal with the integration satisfactorily. However, the limits to be used posed more of a problem. Many pairs of number could yield the correct answer, but the vast majority of candidates did not interpret what they had obtained to give the required area. The question also asked for each loop and so just obtaining the area for one loop correctly did not fully answer the question.

Exemplar 1

5(c)

$$A = \frac{1}{2}\int_0^{2\pi} r^2\,d\theta = \int 9 - 18\sin 2\theta + 9\sin^2 2\theta \; d\theta$$

$$\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$$
$$= 1 - 2\sin^2\theta$$
$$\sin^2 2\theta = \frac{1 - \cos 4\theta}{2}$$

$$\frac{1}{2}\int 9 - 18\sin 2\theta + \frac{9}{2} - \frac{9}{2}\cos 4\theta \; d\theta$$

$$= \frac{1}{2}\left[\frac{27}{2}x + 9\cos 2\theta - \frac{9}{2}\sin 4\theta\right]_0^{2\pi}$$

$$= \frac{27\pi + 9 - 0 \mp 0 - 9}{2} = \frac{27\pi}{2}$$

This is typical of a response that is well laid out. Limits of 0 and 2π are used without explanation which indicated that the candidate may not have fully understood what they needed to find. These limits find the total area of both loops so now two things need to be stated for the last mark, (i) that it does represent both loops and (ii) because of symmetry each loop is half of the given answer.

Question 6

6 Let $y = x\cosh x$.

Prove by induction that, for all integers $n \geqslant 1$, $\dfrac{\mathrm{d}^{2n-1}y}{\mathrm{d}x^{2n-1}} = x\sinh x + (2n-1)\cosh x$. **[6]**

Most candidates understood the principles of induction and knew what to do, proceeding satisfactorily through the necessary steps. However, many of the steps were incomplete. For instance, candidates took as their base case *n* = 1 and differentiated the function correctly but did not show that the result of differentiating to give the left hand side was the same as the right hand side when *n* = 1.

Differentiation had to take place twice and this was satisfactory, but the new result did not relate to the original equation where *k* was replaced by *k* + 1.

Question 7 (a)

7 (a) Determine the values of A, B, C and D such that $\dfrac{x^2+18}{x^2(x^2+9)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$. **[4]**

This part was often well done. However, a number of candidates multiplied throughout the RHS by all three denominators. The fractions on both sides were now not the same unless the numerator of the LHS was multiplied by *x*.

Question 7 (b)

(b) In this question you must show detailed reasoning.

Hence determine the exact value of $\displaystyle\int_3^{\infty} \frac{x^2+18}{x^2(x^2+9)}\,\mathrm{d}x$. **[6]**

There were two issues in this part: (i) The integration had to be carried out successfully using the result of part (a) and this included a $\tan^{-1}$ term. (ii) The correct procedure is now to take the upper limit as some constant value to produce an algebraic value and then to consider what happens to each term as that constant value tends to infinity. Many candidates stated the values as constants rather than showing that the terms tended to constants as *k* tended to infinity.

Exemplar 2

7(b)

$$\int_3^{\infty} \frac{x^2+18}{x^2(x^2+9)}\,dx = \int_3^{\infty} \frac{2}{x^2} - \frac{1}{x^2+9}\,dx$$

$$\int_3^{\infty} 2x^{-2} - \frac{1}{\cancel{x}\,x^2+3^2}\,dx$$

$$\left[-2x^{-1} - \frac{1}{3}\arctan\left(\frac{1}{3}x\right)\right]_3^{t}$$

$$-\frac{2}{t} - \frac{1}{3}\arctan\left(\frac{1}{3}t\right) + \frac{2}{3} + \frac{1}{3}\arctan(1)$$

$$-\frac{2}{t} - \frac{1}{3}\arctan\left(\frac{1}{3}t\right) + \frac{2}{3} + \frac{1}{12}\pi$$

as $t \to \infty$

eq $\to -\frac{1}{6}\pi + \frac{2}{3} + \frac{1}{12}\pi$

$\to \frac{2}{3} - \frac{1}{12}\pi$

Here it can be seen that the upper limit has been replaced by a constant (*t*) and by correct integration the answer is found. Only then is consideration given to each term involving *t* as *t* tends to infinity.

Question 8 (a) (i)

8 A biologist is studying the effect of pesticides on crops. On a certain farm pesticide is regularly applied to a particular crop which grows in soil. Over time, pesticide is transferred between the crop and the soil at a rate which depends on the amount of pesticide in both the crop and the soil. The amount of pesticide in the crop after t days is x grams. The amount of pesticide in the soil after t days is y grams. Initially, when $t = 0$, there is no pesticide in either the crop or the soil.

At first it is assumed that no pesticide is lost from the system. The biologist further assumes that pesticide is added to the crop at a constant rate of k grams per day, where $k > 6$.

After collecting some initial data, the biologist suggests that for $t \geqslant 0$, this situation can be modelled by the following pair of first order linear differential equations.

$$\frac{dx}{dt} = -2x + 78y + k$$
$$\frac{dy}{dt} = 2x - 78y$$

(a) **(i)** Show that $\frac{d^2x}{dt^2} + 80\frac{dx}{dt} = 78k$. **[2]**

The differentiation and substitution was usually done well, though many candidates rather more than they needed to to get to the answer. Those who took the differentiation of x to be 1 rather than $\frac{dx}{dt}$ were unable to complete the question.

Question 8 (a) (ii)

(ii) Determine the particular solution for x in terms of k and t. **[7]**

This question breaks down into a number of steps:

(i) Find the complimentary function.

(ii) Find the particular integral, thus producing a general solution involving 2 arbitrary constants.

(iii) Use the initial values of the question to find the particular solution.

Only a small majority could write out their solution in a logical manner and so got lost at various places

Firstly, the complementary function requires two value of λ one of which in this question was 0. Some therefore ignored this value. Secondly the particular integral could not just be an arbitrary constant because that was included in the complementary function, and this led to errors in the particular solution. Lastly $\frac{dx}{dt}$ was not given to be 0 but had to be worked out to $\frac{dx}{dt} = k$.

A significant number of candidates, however, were able to work through the question to a correct solution.

Exemplar 3

8(a)(ii)

$$\frac{d^2x}{dt^2} + 80\frac{dx}{dt} = 78k \qquad \frac{d^2x}{dt^2} + 80\frac{dx}{dt} = 78k$$

$\lambda^2 + 80\lambda = 0.$

$\lambda = 0, -80.$

$x_c = Ae^{0t} + Be^{-80t} = A + Be^{-80t}.$

PI of the form $pt+q$. $x = pt + q$

$\frac{dx}{dt} = p$

$\frac{d^2x}{dt^2} = 0.$

$80p = 78k.$

$p = \frac{78}{80}k = \frac{39}{40}k.t$

8(a)(ii) (continued)

$\therefore$ Gen solution is $x = A + Be^{-80t} + \frac{39}{40}kt$

$t=0 \quad x=0. \qquad 0 = A + B + \frac{39}{40}kt$

$t=0 \quad \frac{dx}{dt}=0. \qquad \frac{dx}{dt} = -80Be^{-80t} + \frac{39}{40}k.$

$\frac{39}{40}k = 80B$

$\cancel{0 = -80B.}$

$\cancel{B=0}$

$\cancel{A = \frac{39}{40}k.}$

$B = \frac{39}{3200}$

PS:

$x =$

In this response the CF is determined. Then the PI, is given, taking into account the nature of the CF to give the GS. The initial values are then used to find the PS, though this last part has errors.

Question 8 (a) (iii)

If more than 250 grams of pesticide is found in the crop, then it will fail food safety standards.

(iii) The crop is tested 50 days after the pesticide is first added to it.

Explain why, according to this model, the crop will fail food safety standards as a result of this test. **[1]**

The substitution of $t = 50$ gave a value of x in terms of k. When $k = 6$ was substituted a value greater than 250 was achieved with the assumption that if $k > 6$ then an even greater value would result.

Question 8 (b)

Further data collection suggests that some pesticide decays in the soil and so is lost from the system. The model is refined in light of this data. The particular solution for x for this refined model is

$$x = k\left(20 - e^{-41t}\left(20\cosh(\sqrt{1677}\,t) + \frac{819}{\sqrt{1677}}\sinh(\sqrt{1677}\,t)\right)\right).$$

(b) Given now that $k < 12$, determine whether the crop will fail food safety standards in the long run according to this refined model. **[2]**

The vast majority of candidates stated that the second and third terms could be ignored given that e^{-41t} tends to 0. However, this was not quite complete solution because the assertion had to be justified. The crucial point here was that $\sqrt{1677} < 41$ meaning that e^{-41t} dominated the exponential terms of sinh and cosh.

Question 8 (c)

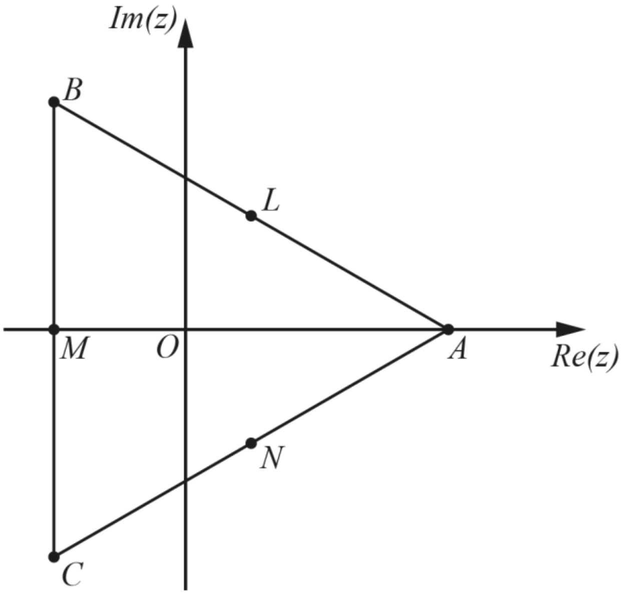
In the refined model, it is still assumed that pesticide is added to the crop at a constant rate.

(c) Suggest a reason why it might be more realistic to model the addition of pesticide as not being at a constant rate. **[1]**

Many reasons were given here, but the important point that had to be made was that in reality the addition of pesticide would not be constant (i.e., continuous) but (possibly) regular. Other reasons such as the changing nature of the crop as it grows, or the weather conditions meant that the farmer might not even apply the pesticide regularly.

Question 9

9 The cube roots of unity are represented on the Argand diagram below by the points A, B and C.



The points L, M and N are the midpoints of the line segments AB, BC and CA respectively.

Determine a degree 6 polynomial equation with integer coefficients whose roots are the complex numbers represented by the points A, B, C, L, M and N. **[5]**

The candidates who realised that the cube roots of unity satisfied the equation $z^3 - 1 = 0$ made the question a little simpler. Those that took the other three points as being the solution to the equation $8z^3 + 1 = 0$ obtained the result in very few lines. However, most candidates gave cartesian coordinates for the 6 points and then multiplied out 6 linear factors of the required equation. This resulted in a lot of algebra, and most candidates that attempted this route lost their way here.

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