Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
In Further Mathematics (8FM0)
Paper 22 Further Pure Mathematics 2

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

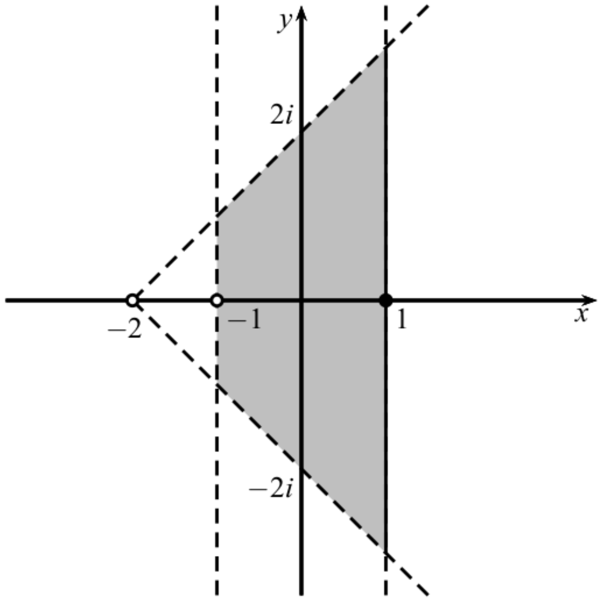
- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme		Marks	AOs
1		Sector from ±2 or ±2i	**M1**	1.1b
		Strip about either axis	**M1**	1.1b
		Correct strip and sector	**A1**	3.1a
		Fully correct	**A1**	2.5
			(4)	
				(4 marks)

Notes:

M1: Draws or identifies a sector, or relevant part thereof, spanning from either ±2 or ±2i.

M1: Either a vertical or horizontal strip drawn or identified, with lines either imaginary or real axis.

A1: Correct sector and strip – so starting sector from −2 on negative real axis, with vertical strip about imaginary axis approximately halfway on real axis between start of sector and *O*. Sector at ±45° (do not worry about the boundary lines for this mark). The strip must be roughly even spaced either side of the imaginary axis, and the sector roughly symmetric in the real axis (allow small tolerance, but A0 if clearly not symmetric)

A1: Inside strip and sector shaded, with correct boundary lines.

Question	Scheme	Marks	AOs
2	$\begin{vmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(-1-\lambda)-6=0$ $\Rightarrow \lambda^2-3\lambda-10=0 \Rightarrow \lambda = ...$	**M1**	3.1a
	$\left(\Rightarrow (\lambda-5)(\lambda+2)=0\right)$ so eigenvalues are −2 and 5	**A1**	1.1b
	For $\lambda=-2$ eigenvector equations are $\begin{cases} 4x+2y=-2x \\ 3x-y=-2y \end{cases} \Rightarrow x, y = ...$ OR For $\lambda=5$ eigenvector equations are $\begin{cases} 4x+2y=5x \\ 3x-y=5y \end{cases} \Rightarrow x, y = ...$	**M1**	2.1
	(For $\lambda=-2$, $3x+y=0$, for $\lambda=5$, $x-2y=0$ so eigenvectors are) One of $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	**A1**	1.1b
	Both of $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	**A1**	1.1b
	Hence e.g. $\mathbf{P} = \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}$ or $\mathbf{P} = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$	**B1ft**	1.1b
	and $\mathbf{D} = \begin{pmatrix} -2 & 0 \\ 0 & 5 \end{pmatrix}$ or $\mathbf{D} = \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$ Note: If **P** is given, their **D** must be consistent with it to award this mark.	**B1ft**	2.2a
		(7)	

(7 marks)

Notes:

M1: Begins the process of finding suitable matrices by attempting the eigenvalues of **M**.
A1: Correct eigenvalues.
M1: Correct method to find an eigenvector for either of their eigenvalues.
A1: One correct eigenvector – accept any non-zero multiples of their eigenvectors.
A1: A correct eigenvector for each of their eigenvalues.
B1ft: Gives **P** as a matrix with their columns as the eigenvectors found.
B1ft: Gives **D** as the matrix with eigenvalues on the diagonal – which must be in the correct order for their **P** (if it is given).

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(i)</td><td>Suppose G has a subgroup of order 11, then (by Lagrange's Theorem) 11 must divide 5291848</td><td>**M1**</td><td>2.1</td></tr>
<tr><td></td><td>But $5-2+9-1+8-4+8=23$</td><td>**M1**</td><td>1.1b</td></tr>
<tr><td></td><td>23 is not divisible by 11, hence 11 does not divide $|G|$, which contradicts Lagrange's Theorem. Hence there is no subgroup of order 11.</td><td>**A1**</td><td>2.4</td></tr>
<tr><td></td><td></td><td>**(3)**</td><td></td></tr>
<tr><td>(ii)(a)</td><td>
<table>
<tr><td>$\times_{30}$</td><td>2</td><td>4</td><td>8</td><td>14</td><td>16</td><td>22</td><td>26</td><td>28</td></tr>
<tr><td>2</td><td>4</td><td>8</td><td>16</td><td>28</td><td>2</td><td>14</td><td>22</td><td>26</td></tr>
<tr><td>4</td><td>8</td><td>**16**</td><td>2</td><td>**26**</td><td>**4**</td><td>28</td><td>14</td><td>**22**</td></tr>
<tr><td>8</td><td>16</td><td>2</td><td>**4**</td><td>**22**</td><td>8</td><td>**26**</td><td>**28**</td><td>14</td></tr>
<tr><td>14</td><td>28</td><td>**26**</td><td>22</td><td>16</td><td>**14**</td><td>8</td><td>4</td><td>**2**</td></tr>
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</table>
Completes at least one row or column correctly
At least 5 rows or columns completed correctly
Completely correct</td><td>**M1**
A1
A1</td><td>1.1b
1.1b
1.1b</td></tr>
<tr><td>(b)</td><td>As the row and column for 16 repeat the borders, 16 is an identity element for $(X, \times_{30})$</td><td>**B1**</td><td>2.2a</td></tr>
<tr><td></td><td>Each element has an inverse as follows:
<table>
<tr><td>x</td><td>2</td><td>4</td><td>8</td><td>14</td><td>16</td><td>22</td><td>26</td><td>28</td></tr>
<tr><td>x^{-1}</td><td>8</td><td>4</td><td>2</td><td>14</td><td>16</td><td>28</td><td>26</td><td>22</td></tr>
</table></td><td>**B1**</td><td>1.1b</td></tr>
<tr><td></td><td>Since we know $\times_{30}$ is associative and as there are no new elements in the table, so $(X, \times_{30})$ is closed, hence $(X, \times_{30})$ is a group.</td><td>**B1**</td><td>2.4</td></tr>
<tr><td></td><td></td><td>**(6)**</td><td></td></tr>
<tr><td colspan="4">**(9 marks)**</td></tr>
</table>

Notes:

(i)

M1: Sets up the proof by stating or implying that if there is a subgroup of order 11 then by Lagrange's Theorem 11 must divide 5291848. May not mention Lagrange's Theorem at this stage. A formal assumption is not required as long as it is implicit.

M1: Applies the divisibility test for 11. Look for an attempt at the alternating sum being used.

A1: Alternating sum is 23, so derives a contradiction as 11 does not divide $|G|$, and conclusion made. Use of Lagrange's Theorem must be clear, though it need not be named.

(ii)(a)

M1: Begins process of completing the table by filling in at least one row or column correctly.

A1: Five or more rows or columns completed correctly.

A1: Completely correct table.

(b)

B1: Identifies 16 as the identity element. No reason needed.

B1: Identifies all inverses or gives reason why each element has an inverse (may refer to each row and column containing the identity once only and symmetrically about the diagonal).

B1: Refers to closure and associativity to deduce $(X, \times_{30})$ is a group.

SC Allow B0B0B1ft for deducing not a group with valid reason if identity or inverse checks fail.

Question	Scheme	Marks	AOs
4(i)(a)	$416 = 5\times72+56$	**M1**	1.1b
	$72 = 1\times56+16;\ 56 = 3\times16+8; 16 = 2\times8(+0)$	**M1**	1.1b
	Hence $h = 8$	**A1**	2.2a
		(3)	
(b)	Using back substitution $8 = 56-3\times16$	**M1**	1.1b
	$= 56-3\left(72-1\times56\right) = 4\times56-3\times72$ $= 4\times\left(416-5\times72\right)-3\times72$	**M1**	1.1b
	$= 4\times416-23\times72$ (so $a = 4$ and $b = -23$)	**A1**	1.1b
		(3)	
(c)	$23\times72 = 4\times416-8 \equiv -8\left(\text{mod}\,416\right)$ Or $23\times72 = 1656 \Rightarrow 1656-k416 = ...$	**M1**	3.1a
	$c = 416-8 = 408$	**A1**	1.1b
		(2)	
(ii)	E.g. $5^{10} \equiv \left(5^2\right)^5 \equiv 25^5 \equiv (-1)^5 \quad \left(\text{mod}\,13\right)$	**M1**	1.1b
	$\equiv -1\left(\text{mod}\,13\right)$	**dM1**	1.1b
	$\equiv 12\left(\text{mod}\,13\right)$	**A1**	2.2a
		(3)	
		(11 marks)	

Notes:

(i) (a)
M1: Starts the process of using the algorithm, with attempt at 416 = $p\times72+q$.
M1: Continues the process until remainder zero is reached.
A1: Deduces the correct highest common factor from correct work.
(b)
M1: Begins the process of back substitution by rearranging their equation with least positive remainder.
M1: Completes the process.
A1: Correct expression or values of a and b identified.
(c)
M1: Uses the answer to (b) and reduces modulo 416 to reach −8 as a congruent number. Or a full process to multiply $23\times72 = 1656$ and reduce by subtracting multiples of 416
A1: For 408.

(ii)
M1: Uses modulo 13 to reduce the equation in some way to reduce a power of 5, e.g. using $5^2 = 25 \equiv -1$ or 12. There will be lots of approaches that can be used here (e.g. $5^3 \equiv 125 \equiv -5\,(\text{mod}\,13)$ is another possibility.

Note the question instructs calculator solutions are not acceptable so finding 5^{10} itself is not acceptable.

dM1: Completes to a smallest (positive or negative) residue – allow if slips, but look for reducing to a number in $\{-6,\ldots 6\}$. Again other routes than the one shown are possible. E.g.

$5^{10} \equiv \left(5^3\right)^3 \times 5 \equiv (-5)^3 \times 5 \equiv -125 \times 5 \equiv 5 \times 5 \equiv 25 \equiv 12 \,(\text{mod}\, 13)$

A1: For 12.

Question	Scheme	Marks	AOs
5(a)	- Immediately after the first tablets are taken there is 20 mg in the person, so $u_0 = 20$ - Reduction by 60% through day means just before next tablet is taken there is $0.4u_n$ mg of the vitamin left in the person - The next tablet taken adds 10 mg to the amount just before the tablet is taken, giving $u_{n+1} = 0.4u_n + 10$	**B1** **B1**	2.4 3.3
		(2)	
(b)	$u_1 = 0.4 \times 20 + 10 = 18$	**B1**	3.1a
	So $\left.\begin{aligned} 20 &= a + b \\ 18 &= \frac{2}{5}a + b \end{aligned}\right\} \Rightarrow 20 - 18 = a\left(1 - \frac{2}{5}\right) \Rightarrow a = \ldots$	**M1**	1.1b
	$a = \frac{10}{3}$ and $b = \frac{50}{3}$	**A1** **A1**	1.1b 1.1b
		(4)	
(c)	In long term $u_n = \frac{10}{3}(0.4)^n + \frac{50}{3} \to \frac{50}{3}$ as $(0.4)^n \to 0$	**M1**	3.4
	Minimum amount of vitamin occurs just before a tablet is taken, so is $\frac{50}{3} - 10 = \frac{20}{3} = 6\tfrac{2}{3}$ mg	**M1**	3.1b
	This is greater than 6 mg and so there is always at least 6 mg of the vitamin in the person. The course of vitamin will be effective.	**A1**	3.2a
		(3)	
			(9 marks)

Notes:

(a)

B1: For explaining any two of the three aspects in the scheme.

B1: All three aspects explained.

(b)

B1: For $u_1 = 18$ found in order to be able to form and solve simultaneous equations.

M1: Setting up and solving equations for *a* and *b* using their u_1 and 20, with at least one value found.

A1: Either *a* or *b* correct.

A1: Both *a* and *b* correct.

(c)

M1: Uses the model to work out the long-term behaviour or the amount of vitamin in the person. Accept identifying $\frac{50}{3}$ as a lower limit for u_n.

M1: Finds the lower bound for the amount of vitamin in the person, subtracting 10 from their long-term value or multiplying the $\frac{50}{3}$ limit by 0.4 to find amount just before next tablet.

A1: Concludes the course of vitamin will be effective following correct work and reasoning.

NB: Attempts that set $u_n = 6$ and try to solve for *n* score no marks. Attempts that set $u_n - 10 = 6$ and try to solve for *n* can score the second M only. Long term behaviour needs to be considered for first M.

Alt (b)	$n = 0 \Rightarrow 20 = a + b$	**B1**	3.1a
	$u_{n+1} = 0.4u_n + 10 \Rightarrow a(0.4)^{n+1} + b = 0.4\left(a(0.4)^n + b\right) + 10$ $\Rightarrow b = 0.4b + 10$	**M1**	1.1b
	$b = \frac{50}{3}$ and $a = \frac{10}{3}$	**A1** **A1**	1.1b 1.1b
		(4)	

Alt (b) notes

B1: Uses *n* = 0 to form a correct relation relating *a* and *b*.

M1: Substitutes the given general form into the recurrence relation and extracts an equation in *b* only.

A1: Correct value for *b*

A1: Both *a* and *b* correct.

Alt 2 (b)	Particular solution is $k \Rightarrow k = 0.4k + 10 \Rightarrow k = \frac{50}{3}$	**B1**	1.ba
	$u_n = a\left(0.4\right)^n + \frac{50}{3} \Rightarrow 20 = a\left(0.4\right)^0 + \frac{50}{3}$	**M1** **A1ft**	3.1a 1.1b
	$a = \frac{10}{3}$	**A1**	1.1b
		(4)	

Alt 2 (b) notes

B1: Uses particular solution is constant to find *b*

M1: Substitutes for *n* =0 or *n* = 1 and sets equal to 20 in the general form to form an equation in *a*

A1: Correct equation follow through their *b*.

A1: *a* correct.

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