# AS LEVEL <br> FURTHER MATHEMATICS 

7366/1 Paper 1
Report on the Examination

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#### Abstract

General

This paper was accessible to the vast majority of students, with most able to demonstrate knowledge in all questions. Poor presentation was an issue for a minority, with some misreading their own writing. Most used a ruler and/or compasses when appropriate, but a few risked drawing circles or straight lines freehand. A significant minority of students approximated their answers unnecessarily, although in most cases they sensibly provided the exact answer also. Many students demonstrated a high level of knowledge and understanding of the mathematics involved and were able to express their methods clearly. Unfortunately, many students were unable to construct a logically ordered proof, instead providing clues scattered around the page for the examiner to find. Some potentially good proofs failed to achieve full marks due to the conclusion being omitted. Students should know that a tick is not typically acceptable as a conclusion.


## Question 1

This question provided a good start, with almost all students recognising the correct response.

## Question 2

This question was answered well by the vast majority of students. A very small minority mistakenly chose the sum of roots $(-p)$ rather than the product of roots.

## Question 3

The majority of students recognised the transformation as a reflection, but a small minority opted for the $x=0$ plane instead of $y=0$

## Question 4

Most students were able to identify the correct product expression, but a significant minority opted for $6(\cos (\alpha \beta)+\mathrm{i} \sin (\alpha \beta))$ instead.

## Question 5

The vast majority of students were able to expand the expression correctly, showing sufficient evidence of their working. Unfortunately, a significant minority were unable to write their solution in sequential lines, instead opting for working or multiplication grids scattered around the page. In most cases sufficient evidence was provided, and so full marks could still be awarded. Most students opted to expand $(2+\mathrm{i})^{2}$ first and then multiply the result by $(2+\mathrm{i})$, but a few used the binomial expansion.

## Question 6

The vast majority of students were able to correctly answer parts (a) and (b). A common incorrect response in part (a) was $\frac{1}{26}$, confusing the determinant with its reciprocal, as used when finding the inverse matrix.

In part (c), many students calculated $\mathbf{B}$ by multiplying $\mathbf{A B}$ by $\mathbf{A}^{-1}$. Most of these attempts correctly pre-multiplied, although a significant minority post-multiplied. Many students failed to see the link between parts (b) and (c), opting instead to form and solve four simultaneous equations from $\left[\begin{array}{cc}5 & 2 \\ -3 & 4\end{array}\right]\left[\begin{array}{ll}a & b \\ c & d\end{array}\right]=\left[\begin{array}{cc}9 & 6 \\ 5 & 12\end{array}\right]$. Most of these attempts were successful, but many of them included an arithmetic error. Some students failed to add their $\mathbf{B}$ to $\mathbf{2 A}$ to complete their answer. Only a very small number of students used the alternative approach involving $2 \mathbf{A}^{2}+\mathbf{A B}$

## Question 7

Most students performed well in this question, with almost all scoring at least one mark in each part. A number of good attempts at parts (a) and/or (b) failed to provide a conclusion. For example, in part (a), most students correctly calculated the value of $\lambda$ for each component, but a small minority did not state the significance of the values being equal.

In part (b), it was sufficient to demonstrate that the scalar product was zero, although some went on to show why a product of zero proves that the angle between the lines is $90^{\circ}$. A small minority used the formula $\cos \theta=\frac{\mathbf{a} \mathbf{b}}{a b}$, unnecessarily calculating the magnitudes of the direction vectors. However, these were generally correct and so were awarded full marks. There were few errors, but a common error was to multiply corresponding components to form a new vector from the products, ie $\left[\begin{array}{c}9 \\ -8 \\ -1\end{array}\right]$ and then state that it equalled zero!

Most students correctly formed three equations in part (c) and attempted to solve two of them. Common errors included misinterpreting the answer by swapping the values of $\lambda$ and $\mu$ when using the calculator's simultaneous equation solver.

It was not uncommon for part (d) answers to be left in position vector form rather than written as bracketed coordinates. However this was condoned on this occasion.

## Question 8

All three parts were answered well. In part (a), some students failed to show any evaluation of the expression, but most clearly demonstrated why the given point lies on the curve. It was essential to show the value of $\cos \frac{\pi}{3}$ as $\frac{1}{2}$. The majority of students realised that the required point in part (b) occurs when $\cos \theta$ takes its minimum value of -1 , and then correctly calculated one of the polar coordinates. However, some students forgot, or were unable, to calculate the other coordinate. In part (c), the vast majority of students correctly calculated the value of $r$, and most of these went on to calculate at least one of the Cartesian coordinates. Some marks were unnecessarily lost due to ambiguous working or inaccurate notation such as $\frac{\sqrt{3}}{4}$ instead of $\sqrt{\frac{3}{4}}$

## Question 9

The majority of students could correctly prove the required identity in part (a). Common errors included $\ln r+\ln 2-\ln r$ or $\frac{\ln (r+2)}{\ln r}$
Most students were able to make some progress in part (b), although only about half were able to see the proof through to its correct conclusion. Common errors included missing brackets or continuing the series to $\ln (n+4)-\ln (n+2)$ or $\ln (n+3)-\ln (n+1)$. The negative signs caused some problems, often leading to $-\ln \frac{1}{2}$ instead of $+\ln \frac{1}{2}$

## Question 10

The vast majority of students were able to write the correct ellipse equation. Common incorrect ellipse equations included $\frac{x^{2}}{4}+\frac{y^{2}}{9}=1$ or $\frac{x^{2}}{3}+\frac{y^{2}}{2}=1$, whilst some common non-ellipse equations included $\frac{x^{2}}{9}-\frac{y^{2}}{4}=1$ and $\frac{x^{2}}{9}+\frac{y^{2}}{4}=0$
Less than half of the students were able to correctly calculate the required volume. Most were able to correctly rearrange the ellipse equation and form an integral expression. Common incorrect rearrangements included $1-\frac{4 x^{2}}{9}$ and $4-\frac{x^{2}}{9}$

## Question 11

This question was a good discriminator, with only a minority of students completing the proof along with a correct conclusion. Almost all solutions dealt with the $n=1$ case first, although not all included a conclusion. Students who incorrectly simplified $\mathbf{A B A}^{-1}$ to $\mathbf{B}$ were unable to score any marks. Another common misconception was that $\left(\mathbf{A B A}^{-1}\right)^{k}$ could be rewritten as $\mathbf{A}^{\mathrm{k}} \mathbf{B}^{\mathrm{k}} \mathbf{A}^{-k}$. Some incorrect proofs included $n=n$, presumably misunderstanding the difference between a variable and a fixed value. Some conclusions included an incorrect statement that it had been proved true for $n=1, n=$ $k$ and $n=k+1$

## Question 12

Parts (a) and (b) were generally answered well. Freehand circles and lines were common, but too ambiguous in a few cases. Almost all students drew a circle with a radius of 2, and most of these were centred on 2i. A common error in part (b) was to extend the half-line into the third quadrant.

The majority of students were unable to make any progress in part (c), and only the strongest students correctly calculated the required magnitude. The most efficient methods employed trigonometry on a suitable triangle. Only a small minority of students recognised that the 'angle in a semicircle' theorem could be used. The most common method was to write the Cartesian equations of the circle and line and then solve them simultaneously.

## Question 13

Almost all students were awarded at least one mark in this question. The majority were able to correctly identify both asymptotes in part (a).

Most students drew the asymptotes in part (b), which helped them to demonstrate the asymptotic behaviour of the curve.

The inequality in part (c) was correctly solved by only half of the students, but most were able to find either one part of the solution or the critical values.

Only a few students were able to find the correct equation in part (d). A common error was to change either $x$ to $-y$ or $y$ to $-x$ but not both. A minority of students wrote down the axis intercepts and asymptotes of the new curve, from which the equation of the new hyperbola could be derived.

## Question 14

Most students realised that the only asymptote had to be horizontal as the degree of the numerator was equal to the degree of the denominator, and so correctly identified its equation. Some incorrect responses included $y=0$ and $x=1$.

Part (b) was a good discriminator. The majority of students realised that the denominator could not be zero, but only about half used its discriminant to find the range of $p$ values. Some incorrect solutions assumed that $p$ had to be an integer, producing the answer $-5 \leq p \leq 5$.
The majority of students were able to find at least one of the axis intercepts in part (c), but only about half considered both axes. Some common errors included use of the denominator when calculating the $x$-intercept, or omitting the negative root.

Most students made some progress in part (d)(i). Common errors included equals signs in place of inequalities, or vice versa.

In part (e), many students realised they had to solve their inequality, but many of these failed to correctly identify the minimum value. Poor notation in writing the exact value of the $y$-coordinate sometimes led to an incorrect answer.

## Question 15

Only about half of the students were able to correctly explain the error in part (a). These often included excellent explanations demonstrating a good understanding of the question.

In part (b), most students solved the quadratic equation, but only about half of them went on to successfully determine the correct sum in the required form. Some attempts used the exponential definitions of sinh and cosh, but these were rarely successful.

## Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results Statistics page of the AQA Website.

