Mark Scheme (Results)

October 2021

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 02 Pure Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$16+(21-1)\times d=24\Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\}\Rightarrow S_{500}=\frac{1}{2}\times 500\{2\times 16+499\times "0.4"\}$	M1	1.1b
	$=57900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times "0.4"=215.6\Rightarrow S_{500}=\frac{1}{2}\times 500\{16+"215.6"\}$	M1	1.1b
	$=57900$	A1	1.1b
			(4 marks)

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using *a* = 16, and *n* = 21 and the 24. The method may be implied by their working.
If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20}, \frac{4}{10}, \frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with *a* = 16, *n* = 500 and their numerical *d* from part (a)
If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their *l*

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

(a) $d=\frac{24-16}{21}=\frac{8}{21}$ (b) $S_{500}=\frac{1}{2}\times 500\left\{2\times 16+499\times\frac{8}{21}\right\}=55523.80952...$

This scores (a) M0A0 (b) M1A0

Question	Scheme	Marks	AOs
2(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$\mathrm{f}(1.8) = 7 - 2\times 1.8^2 = 0.52 \Rightarrow \mathrm{gf}(1.8) = \mathrm{g}(0.52) = \dfrac{3\times 0.52}{5\times 0.52 - 1} = ...$	M1	1.1b
	$\mathrm{gf}(1.8) = 0.975$ oe e.g. $\dfrac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \dfrac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(\mathrm{g}^{-1}(x) =\right)\dfrac{x}{5x-3}$	A1	2.2a
		(2)	
			(5 marks)

Notes

(a)

B1: Correct range. Allow f (x) or f for y. Allow e.g. $\{y \in \mathbb{R} : y \leqslant 7\}, -\infty < y \leqslant 7, (-\infty, 7]$

(b)

M1: Full method to find f (1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute x = 1.8 into an attempt at gf (x).

E.g. $\mathrm{gf}(x) = \dfrac{3(7-2x^2)}{5(7-2x^2)-1} = \dfrac{3\left(7-2(1.8)^2\right)}{5\left(7-2\times(1.8)^2\right)-1} = ...$

A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\dfrac{-x}{3-5x}, \dfrac{1}{5} + \dfrac{3}{25x-15}$

Ignore any domain if given.

Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\frac{12y+5}{1-3y}=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
	$y=\frac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

Notes

B1(M1 on EPEN): Applies at least one addition or subtraction law of logs correctly.
Can also be awarded for using $2=\log_3 9$. This may be implied by e.g.
$\log_3 ...=2 \Rightarrow ...=9$

M1: A rigorous argument with no incorrect working to remove the log or logs correctly and obtain a correct equation in any form **and** solve for *y*.

A1: Correct **exact** value. Allow equivalent fractions.

Guidance on how to mark particular cases:

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M1A1

Question	Scheme	Marks	AOs
4	**Examples:** $4\sin\frac{\theta}{2} \approx 4\left(\frac{\theta}{2}\right),\ 3\cos^2\theta \approx 3\left(1-\frac{\theta^2}{2}\right)^2$ $3\cos^2\theta = 3\left(1-\sin^2\theta\right) \approx 3\left(1-\theta^2\right)$ $3\cos^2\theta = 3\frac{(\cos 2\theta+1)}{2} \approx \frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	M1	1.1a
	Examples: $4\sin\frac{\theta}{2}+3\cos^2\theta \approx 4\left(\frac{\theta}{2}\right)+3\left(1-\frac{\theta^2}{2}\right)^2$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\left(\frac{\theta}{2}\right)+3\left(1-\sin^2\theta\right) \approx 2\theta+3\left(1-\theta^2\right)$ $4\sin\frac{\theta}{2}+3\cos^2\theta = 4\sin\frac{\theta}{2}+3\frac{(\cos 2\theta+1)}{2} \approx 4\left(\frac{\theta}{2}\right)+\frac{3}{2}\left(1-\frac{4\theta^2}{2}+1\right)$	dM1	1.1b
	$= 2\theta+3\left(1-\theta^2+\ldots\right) = 3+2\theta-3\theta^2$	A1	2.1
		(3)	
			(3 marks)

Notes

M1: Attempts to use at least one correct approximation **within the given expression**.
Either $\sin\frac{\theta}{2} \approx \frac{\theta}{2}$ or $\cos\theta \approx 1-\frac{\theta^2}{2}$ or e.g. $\sin\theta \approx \theta$ if they write $\cos^2\theta$ as $1-\sin^2\theta$ or e.g. $\cos 2\theta \approx 1-\frac{(2\theta)^2}{2}$ (condone missing brackets) if they write $\cos^2\theta$ as $\frac{1+\cos 2\theta}{2}$.
Allow sign slips only with any identities used but the appropriate approximations must be applied.

dM1: Attempts to use correct approximations **with the given expression** to obtain an expression in terms of θ only. **Depends on the first method mark.**

A1: Correct terms following correct work. Allow the terms in any order and ignore any extra terms if given correct or incorrect.

Question	Scheme	Marks	AOs
5(a)(i)	$\frac{dy}{dx}=20x^3-72x^2+84x-32$	M1 A1	1.1b 1.1b
(ii)	$\frac{d^2y}{dx^2}=60x^2-144x+84$	A1ft	1.1b
		(3)	
(b)(i)	$x=1\Rightarrow\frac{dy}{dx}=20-72+84-32$	M1	1.1b
	$\frac{dy}{dx}=0$ so there is a stationary point at x = 1	A1	2.1
	Alternative for (b)(i)		
	$20x^3-72x^2+84x-32=4(x-1)^2(5x-8)=0\Rightarrow x=...$	M1	1.1b
	When x = 1, $\frac{dy}{dx}=0$ so there is a stationary point	A1	2.1
(b)(ii)	**Note that in (b)(ii) there are no marks for <u>just</u> evaluating** $\left(\frac{d^2y}{dx^2}\right)_{x=1}$		
	E.g. $\left(\frac{d^2y}{dx^2}\right)_{x=0.8}=...\quad\left(\frac{d^2y}{dx^2}\right)_{x=1.2}=...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=0.8}>0,\quad\left(\frac{d^2y}{dx^2}\right)_{x=1.2}<0$ Hence point of inflection	A1	2.2a
		(4)	
	Alternative 1 for (b)(ii)		
	$\left(\frac{d^2y}{dx^2}\right)_{x=1}=60x^2-144x+84=0$ (is inconclusive) $\left(\frac{d^3y}{dx^3}\right)=120x-144\Rightarrow\left(\frac{d^3y}{dx^3}\right)_{x=1}=...$	M1	2.1
	$\left(\frac{d^2y}{dx^2}\right)_{x=1}=0$ and $\left(\frac{d^3y}{dx^3}\right)_{x=1}\neq0$ Hence point of inflection	A1	2.2a
	Alternative 2 for (b)(ii)		
	E.g. $\left(\frac{dy}{dx}\right)_{x=0.8}=...\quad\left(\frac{dy}{dx}\right)_{x=1.2}=...$	M1	2.1
	$\left(\frac{dy}{dx}\right)_{x=0.8}<0,\quad\left(\frac{dy}{dx}\right)_{x=1.2}<0$ Hence point of inflection	A1	2.2a
			(7 marks)

Notes

(a)(i)
M1: $x^n\rightarrow x^{n-1}$ for at least one power of x
A1: $\frac{dy}{dx}=20x^3-72x^2+84x-32$
(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 20x^3 - 72x^2 + 84x - 32$

(b)(i)

M1: Substitutes $x = 1$ into their $\frac{dy}{dx}$

A1: Obtains $\frac{dy}{dx} = 0$ following a correct derivative and makes a conclusion which can be minimal e.g. tick, QED etc. which may be in a preamble e.g. stationary point when $\frac{dy}{dx} = 0$ and then shows $\frac{dy}{dx} = 0$

Alternative:

M1: Attempts to solve $\frac{dy}{dx} = 0$ by factorisation. This may be by using the factor of (x – 1) or possibly using a calculator to find the roots and showing the factorisation. Note that they may divide by 4 before factorising which is acceptable. Need to either see either $4(x-1)^2(5x-8)$ or $(x-1)^2(5x-8)$ for the factorisation or $x = \frac{8}{5}$ and $x = 1$ seen as the roots.

A1: Obtains $x = 1$ and makes a conclusion as above

(b)(ii)

M1: Considers the value of the second derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

(NB $\frac{d^2y}{dx^2} = (x-1)(60x-84)$ so may use this factorised form when considering $x < 1$, $x > 1$ for sign change of second derivative)

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that the stationary point is a point of inflection. Sufficient reason is e.g. "sign change"/ "> 0, < 0". If values are given they should be correct (but be generous with accuracy) but also just allow "> 0" and "< 0" provided they are correctly paired. The interval must be where $x < 1.4$

Alternative 1 for (b)(ii)

M1: Shows that second derivative at $x = 1$ is zero and **then finds the third derivative at $x = 1$**

A1: Fully correct work including a correct $\frac{d^2y}{dx^2}$ with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is "$\neq 0$" but must follow a correct third derivative and a correct value if evaluated. For reference $\left(\frac{d^3y}{dx^3}\right)_{x=1} = -24$

Alternative 2 for (b)(ii)

M1: Considers the value of the first derivative either side of $x = 1$. Do not be too concerned with the interval for the method mark.

A1: Fully correct work with a reasoned conclusion indicating that stationary point is a point of inflection. Sufficient reason is e.g. "same sign"/"both negative"/"< 0, < 0". If values are given they should be correct (but be generous with accuracy). The interval must be where $x < 1.4$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
f′(x)	-32	-24.3	-17.92	-12.74	-8.64	-5.5	-3.2	-1.62	-0.64	-0.14	0
f″(x)	84	70.2	57.6	46.2	36	27	19.2	12.6	7.2	3	0

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7
f′(x)	-0.1	-0.32	-0.54	-0.64	-0.5	0	0.98
f″(x)	-1.8	-2.4	-1.8	0	3	7.2	12.6

Question	Scheme	Marks	AOs
6(a)	Angle $AOB = \dfrac{\pi-\theta}{2}$	B1	2.2a
		(1)	
(b)	Area = $2\times\frac{1}{2}r^2\left(\frac{\pi-\theta}{2}\right)+\frac{1}{2}(2r)^2\theta$	M1	2.1
	$=\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta+2r^2\theta=\frac{3}{2}r^2\theta+\frac{1}{2}r^2\pi=\frac{1}{2}r^2(3\theta+\pi)$*	A1*	1.1b
		(2)	
(c)	Perimeter = $4r+2r\left(\frac{\pi-\theta}{2}\right)+2r\theta$	M1	3.1a
	$=4r+r\pi+r\theta$ or e.g. $r(4+\pi+\theta)$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)
B1: Deduces the correct expression for angle *AOB*
Note that $\dfrac{180-\theta}{2}$ scores B0

(b)
M1: Fully correct strategy for the area using their angle from (a) appropriately.
Need to see $2\times\frac{1}{2}r^2\alpha$ or just $r^2\alpha$ where α is their angle in terms of θ from part (a) + $\frac{1}{2}(2r)^2\theta$ with or without the brackets.

A1*: Correct proof. For this mark you can condone the omission of the brackets in $\frac{1}{2}(2r)^2\theta$ as long as they are recovered in subsequent work e.g. when this term becomes $2r^2\theta$
The first term must be seen expanded as e.g. $\frac{1}{2}r^2\pi-\frac{1}{2}r^2\theta$ or equivalent

(c)
M1: Fully correct strategy for the perimeter using their angle from (a) appropriately
Need to see $4r+2r\alpha+2r\theta$ where α is their angle from part (a) in terms of θ
A1: Correct simplified expression

Note that some candidates may change the angle to degrees at the start and all marks are available e.g.

(a) $\dfrac{180-\frac{180\theta}{\pi}}{2}$

(b) $2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\dfrac{1}{360}\times\pi r^2+\dfrac{\theta}{360}\times\dfrac{180}{\pi}\times\pi(2r)^2=\dfrac{1}{2}\pi r^2-\dfrac{1}{2}r^2\theta+2r^2\theta=\dfrac{1}{2}r^2(3\theta+\pi)$

(c) $4r+2\left(\dfrac{180-\frac{180\theta}{\pi}}{2}\right)\times\dfrac{1}{360}\times2\pi r+\dfrac{180\theta}{\pi}\times\dfrac{1}{360}\times2\pi(2r)=4r+\pi r+r\theta$

Question	Scheme	Marks	AOs
7(a)	$y = x^3 - 10x^2 + 27x - 23 \Rightarrow \frac{dy}{dx} = 3x^2 - 20x + 27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5} = 3\times5^2 - 20\times5 + 27 (= 2)$	M1	1.1b
	$y + 13 = 2(x - 5)$	M1	2.1
	$y = 2x - 23$	A1	1.1b
		(4)	
(b)	Both C and l pass through (0, −23) and so C meets l again on the y-axis	B1	2.2a
		(1)	
(c)	$\pm\int \left(x^3 - 10x^2 + 27x - 23 - (2x - 23)\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{25}{2}x^2\right]_0^5$ $= \left(\frac{625}{4} - \frac{1250}{3} + \frac{625}{2}\right)(-0)$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative:		
	$\pm\int \left(x^3 - 10x^2 + 27x - 23\right) dx$ $= \pm\left(\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4} - \frac{10}{3}x^3 + \frac{27}{2}x^2 - 23x\right]_0^5 + \frac{1}{2}\times5(23+13)$ $= -\frac{455}{12} + 90$	dM1	2.1
	$= \frac{625}{12}$	A1	1.1b
			(9 marks)

Notes

(a)
B1: Correct derivative
M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$
M1: Fully correct straight line method using (5, – 13) and their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 5$
A1: cao. Must see the full equation in the required form.
(b)
B1: Makes a suitable deduction.

Alternative via equating l and C and factorising e.g.

$$x^3 - 10x^2 + 27x - 23 = 2x - 23$$
$$x^3 - 10x^2 + 25x = 0$$
$$x\left(x^2 - 10x + 25\right) = 0 \Rightarrow x = 0$$

So they meet on the y-axis

(c)
M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±"$C – l$"
A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a))

If they attempt as 2 separate integrals e.g. $\int\left(x^3 - 10x^2 + 27x - 23\right)\,\mathrm{d}x - \int\left(2x - 23\right)\,\mathrm{d}x$ then award this mark for the correct integration of the curve as in the alternative.
If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for ±"$C – l$"

dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.**
A1: Correct exact value
Alternative:
M1: For an attempt to integrate $x^n \to x^{n+1}$ for ±C
A1: Correct integration for ±C
dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction.

May see the trapezium area attempted as $\int\left(2x - 23\right)\,\mathrm{d}x$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation.

Depends on the first method mark.

A1: Correct exact value

Note if they do $l – C$ rather than $C – l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l – C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks.

If the answer is left as $-\frac{625}{12}$ then score A0

Question	Scheme	Marks	AOs
8(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(3y^2)=6y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}}{\mathrm{d}x}(qxy)=qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy$	M1	2.1
	$3px^2+qx\frac{\mathrm{d}y}{\mathrm{d}x}+qy+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$(qx+6y)\frac{\mathrm{d}y}{\mathrm{d}x}=-3px^2-qy\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=...$	dM1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3px^2-qy}{qx+6y}$	A1	1.1b
		(4)	
(b)	$p(-1)^3+q(-1)(-4)+3(-4)^2=26$	M1	1.1b
	$19x+26y+123=0\Rightarrow m=-\frac{19}{26}$	B1	2.2a
	$\frac{-3p(-1)^2-q(-4)}{q(-1)+6(-4)}=\frac{26}{19}$ or $\frac{q(-1)+6(-4)}{3p(-1)^2+q(-4)}=-\frac{19}{26}$	M1	3.1a
	$p-4q=22,\ 57p-102q=624\Rightarrow\ p=...,q=...$	dM1	1.1b
	$p=2,\ \ q=-5$	A1	1.1b
		(5)	
		(9 marks)	

Notes

(a)

M1: For selecting the appropriate method of differentiating:

Allow this mark for either $3y^2\to\alpha y\frac{\mathrm{d}y}{\mathrm{d}x}$ or $qxy\to\alpha x\frac{\mathrm{d}y}{\mathrm{d}x}+\beta y$

A1: Fully correct differentiation. Ignore any spurious $\frac{\mathrm{d}y}{\mathrm{d}x}=...$

dM1: A valid attempt to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject with 2 terms only in $\frac{\mathrm{d}y}{\mathrm{d}x}$ coming from qxy and $3y^2$

Depends on the first method mark.

A1: Fully correct expression

(b)

M1: Uses $x=-1$ and $y=-4$ in the equation of C to obtain an equation in p and q

B1: Deduces the correct gradient of the given normal.

This may be implied by e.g.

$19x+26y+123=0\Rightarrow y=-\frac{19}{26}x+...\Rightarrow$ Tangent equation is $y=\frac{26}{19}x+...$

M1: Fully correct strategy to establish an equation connecting p and q using $x=-1$ and $y=-4$ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the gradient of the normal. E.g. $(a)=-1\div\text{their}-\frac{19}{26}$ or $-1\div(a)=\text{their}-\frac{19}{26}$

dM1: Solves simultaneously to obtain values for p and q.

Depends on both previous method marks.

A1: Correct values

Alternative for (b):

$$\frac{dy}{dx}=\frac{-3p+4q}{-q-24}\Rightarrow y+4=\frac{q+24}{4q-3p}(x+1)$$

M1A1

$$\Rightarrow y(4q-3p)+4(4q-3p)=(q+24)x+q+24$$

M1

$$19x+26y+123=0\Rightarrow q+24=19\Rightarrow q=-5$$

$$3p-4q=26\Rightarrow 3p+20=26\Rightarrow p=2$$

M1A1

M1: Uses (–1, –4) in the tangent gradient and attempts to form normal equation
A1: Correct equation for normal
M1: Multiplies up so that coefficients can be compared
dM1: Full method comparing coefficients to find values for p and q
A1: Correct values

Question	Scheme	Marks	AOs
9	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\dfrac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=\ldots$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\dfrac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=\ldots$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+\ldots=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-\ldots=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-\ldots$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\ldots\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=\ldots$	M1	3.1a
	$=\frac{9}{28}$ *	A1*	1.1b
			(3 marks)

Notes

B1: Deduces the correct value of the **first** term or the common ratio. The correct first term can be seen as part of them writing down the sequence but must be the **first** term.

M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with $a = \frac{9}{16}$ and $r = \pm\frac{3}{4}$

A1*: Correct proof

Alternative 1:

B1: Deduces the correct value for the sum to infinity (starting at n = 1) or the common ratio

M1: Calculates the required value by subtracting the first term from their sum to infinity

A1*: Correct proof

Alternative 2:

B1: Deduces the correct value of the **first** term or the common ratio.

M1: Splits the series into "odds" and "evens", attempts the sum of both parts and calculates the required value by adding both sums

A1*: Correct proof

Alternative 3:

B1: Deduces the correct value of the **first** term

M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for "S"

A1*: Correct proof

Question	Scheme	Marks	AOs
10(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
		(6 marks)	

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow *t* rather than *T* and *A* rather than *a*.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$

$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b \ *$$

M1: Uses the given answer and uses the power law and addition law correctly

A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for *b* (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of *a*.

E.g. substitutes one of the given points and their value for *b* into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of *a*. Allow slips in rearranging their equation but must be correct log work to find *a*.

Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of *a*.

E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$

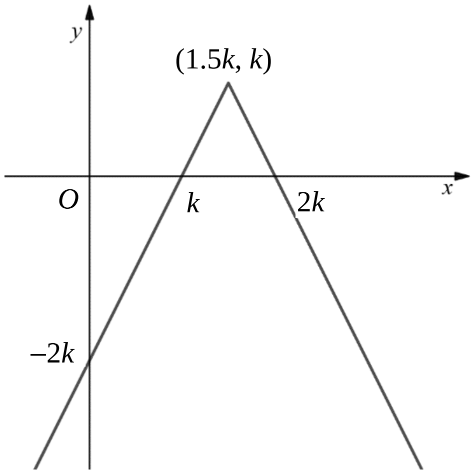
A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$

(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the <u>equation</u> not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation

Question	Scheme	Marks	AOs
11(a)			
	$\wedge$ shape in any position	B1	1.1b
	Correct x-intercepts or coordinates	B1	1.1b
	Correct y-intercept or coordinates	B1	1.1b
	Correct coordinates for the vertex of a $\wedge$ shape	B1	1.1b
		(4)	
(b)	$x = k$	B1	2.2a
	$k-(2x-3k)=x-k \Rightarrow x=...$	M1	3.1a
	$x=\frac{5k}{3}$	A1	1.1b
	Set notation is required here for this mark $\left\{x : x<\frac{5k}{3}\right\} \cap \{x : x>k\}$	A1	2.5
		(4)	
(c)	$x = 3k$ **or** $y = 3 - 5k$	B1ft	2.2a
	$x = 3k$ **and** $y = 3 - 5k$	B1ft	2.2a
		(2)	
			(10 marks)

Notes

(a) **Note that the sketch may be seen on Figure 4**
B1: See scheme
B1: Correct x-intercepts. Allow as shown or written as $(k, 0)$ and $(2k, 0)$ and condone coordinates written as $(0, k)$ and $(0, 2k)$ as long as they are in the correct places.
B1: Correct y-intercept. Allow as shown or written as $(0, -2k)$ or $(-2k, 0)$ as long as it is in the correct place. Condone $k - 3k$ for $-2k$.
B1: Correct coordinates as shown
Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as $y = 0$, $x = k$ etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.
(b)
B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$
M1: Attempts to solve $k - (2x - 3k) = x - k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching $k = …$ or $x = …$ as long as they are solving the required equation.
A1: Correct value
A1: Correct answer using the correct set notation.

Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x :\ k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow "|" for " :"

But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\left\{x :\ k < x,\ x < \frac{5k}{3}\right\}$ scores A0

(c)

B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..$, $y = \ldots$

B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"k" but must be in terms of k.

Allow as coordinates or $x = ..$, $y = \ldots$

If coordinates are given the wrong way round and not seen correctly as $x = \ldots$, $y = \ldots$

e.g. $(3 - 5k, 3k)$ this is B0B0

Question	Scheme	Marks	AOs
12(a)	$u=1+\sqrt{x} \Rightarrow x=(u-1)^2 \Rightarrow \frac{dx}{du}=2(u-1)$ or $u=1+\sqrt{x} \Rightarrow \frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$	B1	1.1b
	$\int \frac{x}{1+\sqrt{x}}dx=\int \frac{(u-1)^2}{u}2(u-1)du$ or $\int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times 2x^{\frac{1}{2}}du=\int \frac{2x^{\frac{3}{2}}}{u}du=\int \frac{2(u-1)^3}{u}du$	M1	2.1
	$\int_0^{16} \frac{x}{1+\sqrt{x}}dx=\int_1^5 \frac{2(u-1)^3}{u}du$	A1	1.1b
		(3)	
(b)	$2\int \frac{u^3-3u^2+3u-1}{u}du=2\int \left(u^2-3u+3-\frac{1}{u}\right)du=...$	M1	3.1a
	$=(2)\left[\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u\right]$	A1	1.1b
	$=2\left[\frac{5^3}{3}-\frac{3(5)^2}{2}+3(5)-\ln 5-\left(\frac{1}{3}-\frac{3}{2}+3-\ln 1\right)\right]$	dM1	2.1
	$=\frac{104}{3}-2\ln 5$	A1	1.1b
		(4)	
		(7 marks)	

Notes

(a)

B1: Correct expression for $\frac{dx}{du}$ or $\frac{du}{dx}$ $(\text{or } u')$ or dx in terms of du or du in terms of dx

M1: Complete method using the given substitution.

This needs to be a correct method for their $\frac{dx}{du}$ or $\frac{du}{dx}$ leading to an integral in terms of u only (ignore any limits if present) so for each case you need to see:

$$\frac{dx}{du}=f(u) \to \int \frac{x}{1+\sqrt{x}}dx=\int \frac{(u-1)^2}{u}f(u)du$$

$\frac{du}{dx}=g(x) \to \int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times\frac{du}{g(x)}=\int h(u)du$. In this case you can condone slips with coefficients e.g. allow $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}} \to \int \frac{x}{1+\sqrt{x}}dx=\int \frac{x}{u}\times\frac{x^{\frac{1}{2}}}{2}du=\int h(u)du$

but not $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \rightarrow \int \frac{x}{1+\sqrt{x}}\mathrm{d}x = \int \frac{x}{u} \times \frac{x^{-\frac{1}{2}}}{2}\mathrm{d}u = \int \mathrm{h}(u)\mathrm{d}u$

A1: All correct with correct limits and no errors. The "du" must be present but may have been omitted along the way but it must have been seen at least once before the final answer. The limits can be seen as part of the integral or stated separately.

(b)

M1: Realises the requirement to cube the bracket and divide through by u and makes progress with the integration to obtain at least 3 terms of the required form e.g. 3 from ku^3, ku^2, ku, $k\ln u$

A1: Correct integration. This mark can be scored with the "2" still outside the integral or even if it has been omitted. But if the "2" has been combined with the integrand, the integration must be correct.

dM1: Completes the process by applying their "changed" limits and subtracts the right way round **Depends on the first method mark.**

A1: Cao (Allow equivalents for $\frac{104}{3}$ e.g. $\frac{208}{6}$)

Question	Scheme	Marks	AOs
13(a)	$y=\text{cosec}^3\theta \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}\theta}=-3\text{cosec}^2\theta\text{cosec}\,\theta\cot\theta$	B1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\div\frac{\mathrm{d}x}{\mathrm{d}\theta}$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3\text{cosec}^3\theta\cot\theta}{2\cos 2\theta}$	A1	1.1b
		(3)	
(b)	$y=8\Rightarrow \text{cosec}^3\theta=8\Rightarrow \sin^3\theta=\frac{1}{8}\Rightarrow \sin\theta=\frac{1}{2}$	M1	3.1a
	$\theta=\frac{\pi}{6}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3\text{cosec}^3\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)}{2\cos\left(\frac{2\pi}{6}\right)}=...$ **or** $\sin\theta=\frac{1}{2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{-3}{\sin^3\theta}\times\frac{\cos\theta}{\sin\theta}}{2\left(1-2\sin^2\theta\right)}=\frac{-3\times 8\times\frac{\sqrt{3}/2}{1/2}}{2\left(1-2\times\frac{1}{4}\right)}$	M1	2.1
	$=-24\sqrt{3}$	A1	2.2a
		(3)	
			(6 marks)

Notes

(a)

B1: Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ seen or implied in any form e.g. $\frac{-3\cos\theta}{\sin^4\theta}$

M1: Obtains $\frac{\mathrm{d}x}{\mathrm{d}\theta}=k\cos 2\theta$ or $\alpha\cos^2\theta+\beta\sin^2\theta$ (from product rule on sinθcosθ)

and attempts $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\div\frac{\mathrm{d}x}{\mathrm{d}\theta}$

A1: Correct expression in any form.

May see e.g. $\frac{-3\cos\theta}{2\sin^4\theta\cos 2\theta},\ -\frac{3}{4\sin^4\theta\cos\theta-2\sin^3\theta\tan\theta}$

(b)

M1: Recognises the need to find the value of $\sin\theta$ or θ when y = 8 and uses the y parameter to establish its value. This should be correct work leading to $\sin\theta=\frac{1}{2}$ or e.g. $\theta=\frac{\pi}{6}$ or 30°.

M1: Uses their value of $\sin\theta$ or θ in their $\frac{\mathrm{d}y}{\mathrm{d}x}$ from part (a) (working in exact form) in an attempt to obtain an exact value for $\frac{\mathrm{d}y}{\mathrm{d}x}$. May be implied by a correct exact answer.

If no working is shown but an exact answer is given you may need to check that this follows their $\frac{\mathrm{d}y}{\mathrm{d}x}$.

A1: Deduces the correct gradient

Question	Scheme	Marks	AOs
14(a)	$\frac{\mathrm{d}V}{\mathrm{d}t}=0.48-0.1h$	B1	3.1b
	$V=24h \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}h}=24$ or $\frac{\mathrm{d}h}{\mathrm{d}V}=\frac{1}{24}$	B1	3.1b
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}V}=\frac{0.48-0.1h}{24}$ or e.g. $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow 0.48-0.1h=24\frac{\mathrm{d}h}{\mathrm{d}t}$	M1	2.1
	$1200\frac{\mathrm{d}h}{\mathrm{d}t}=24-5h$*	A1*	1.1b
		(4)	
(b)	$1200\frac{\mathrm{d}h}{\mathrm{d}t}=24-5h \Rightarrow \int\frac{1200}{24-5h}\mathrm{d}h=\int\mathrm{d}t$ $\Rightarrow$ *e.g.* $\alpha\ln(24-5h)=t(+c)$ oe or $1200\frac{\mathrm{d}h}{\mathrm{d}t}=24-5h \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}h}=\frac{1200}{24-5h}$ $\Rightarrow$ *e.g.* $t(+c)=\alpha\ln(24-5h)$ oe	M1	3.1a
	$t=-240\ln(24-5h)(+c)$ oe	A1	1.1b
	$t=0, h=2 \Rightarrow 0=-240\ln(24-10)+c \Rightarrow c=...(240\ln 14)$	M1	3.4
	$t=240\ln(14)-240\ln(24-5h)$	A1	1.1b
	$t=240\ln\frac{14}{24-5h}\Rightarrow\frac{t}{240}=\ln\frac{14}{24-5h}\Rightarrow e^{\frac{t}{240}}=\frac{14}{24-5h}$ $\Rightarrow 14e^{-\frac{t}{240}}=24-5h\Rightarrow h=...$	ddM1	2.1
	$h=4.8-2.8e^{-\frac{t}{240}}$ oe e.g. $h=\frac{24}{5}-\frac{14}{5}e^{-\frac{t}{240}}$	A1	3.3
		(6)	
(c)	**Examples:** • As $t\to\infty, e^{-\frac{t}{240}}\to 0$ • When h > 4.8, $\frac{\mathrm{d}V}{\mathrm{d}t}<0$ • Flow in = flow out at max h so 0.1h = 4.8 → h = 4.8 • As $e^{-\frac{t}{240}}>0,\ h<4.8$ • $h=5\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}t}=-0.02$ or $\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{1200}$ • $\frac{\mathrm{d}h}{\mathrm{d}t}=0\Rightarrow h=4.8$ • $h=5\Rightarrow 4.8-2.8e^{-\frac{t}{240}}=5\Rightarrow e^{-\frac{t}{240}}<0$	M1	3.1b
	• The limit for h (according to the model) is 4.8m and the tank is 5m high so the tank will never become full • If h = 5 the tank would be emptying so can never be full • The equation can't be solved when h = 5	A1	3.2a

		(2)	
			(12 marks)

Notes

(a)

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}t}$ according to the model

B1: Identifies the correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$ according to the model

M1: Applies $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}V}$ or equivalent correct formula with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}h}$ which may be implied by their working

A1*: Correct equation obtained with no errors

Note that: $\frac{\mathrm{d}V}{\mathrm{d}t}=0.48-0.1h \Rightarrow \frac{\mathrm{d}h}{\mathrm{d}t}=\frac{0.48-0.1h}{24} \Rightarrow 1200\frac{\mathrm{d}h}{\mathrm{d}t}=24-5h$* scores B1B0M0A0. There must be clear evidence where the "24" comes from and evidence of the correct chain rule being applied.

(b)

M1: Adopts a correct strategy by separating the variables correctly or rearranges to obtain $\frac{\mathrm{d}t}{\mathrm{d}h}$ correctly in terms of h **and** integrates to obtain $t=\alpha\ln(24-5h)(+c)$ or equivalent (condone missing brackets around the "24 – 5h") and + c not required for this mark.

A1: Correct equation in any form and + c not required. Do not condone missing brackets unless they are implied by subsequent work.

M1: Substitutes t = 0 and h = 2 to find their constant of integration (there must have been some attempt to integrate)

A1: Correct equation in any form

ddM1: Uses fully correct log work to obtain h in terms of t.

This depends on <u>both</u> previous method marks.

A1: Correct equation

Note that the marks may be earned in a different order e.g.:

$$t+c=-240\ln(24-5h) \Rightarrow -\frac{t}{240}+d=\ln(24-5h) \Rightarrow A\mathrm{e}^{-\frac{t}{240}}=24-5h$$

$$t=0, h=2 \Rightarrow A=14 \Rightarrow 14\mathrm{e}^{-\frac{t}{240}}=24-5h \Rightarrow h=4.8-2.8\mathrm{e}^{-\frac{t}{240}}$$

Score as M1 A1 as in main scheme then

M1: Correct work leading to $A\mathrm{e}^{\alpha t}=24-5h$ (must have a constant "A")

A1: $A\mathrm{e}^{-\frac{t}{240}}=24-5h$

ddM1: Uses t = 0, h = 2 in an expression of the form above to find A

A1: $h=4.8-2.8\mathrm{e}^{-\frac{t}{240}}$

(c)

M1: See scheme for some examples

A1: Makes a correct interpretation for their method.
There must be no incorrect working or contradictory statements.
This is not a follow through mark and if their equation in (b) is used it must be correct.

Question	Scheme	Marks	AOs
15(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
		(3)	
(b)(i)	$3+2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$ $\Rightarrow t=...$	M1	3.4
	t = 11.6	A1	1.1b
		(3)	
(c)	$3+2\sqrt{5}\cos(0.5t+0.464)=0$ $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977...-0.464)-2(2.306...-0.464)$	dM1	3.1b
	= 3.34	A1	1.1b
		(4)	
(d)	e.g. the "3" would need to vary	B1	3.5c
		(1)	
			(11 marks)

Notes

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{1}{2}$ or $\sin\alpha=\pm\frac{1}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha=$awrt 0.464

(b)(i)

B1ft: For $\left(3+2\sqrt{5}\right)$m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their R value so allow 3 + 2×Their R. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t\pm"0.464"=2\pi$ to obtain a value for t

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t\pm"26.6"=360$

A1: Awrt 11.6

Alternative for (b):

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t) \Rightarrow \frac{dH}{dt} = -2\sin(0.5t) - \cos(0.5t) = 0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677..., 5.819... \Rightarrow t = 5.36,\ 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{dH}{dt}$ and attempts to solve $\frac{dH}{dt} = 0$ for t

A1: For t = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their R

(c)

M1: Uses the model and sets $3 + 2"\sqrt{5}"\cos(...) = 0$ and proceeds to $\cos(...) = k$ where $|k| < 1$.

Allow e.g. $3 + 2"\sqrt{5}"\cos(...) < 0$

dM1: Solves $\cos(0.5t \pm "0.464") = k$ where $|k| < 1$ to obtain at least one value for t

This requires e.g. $2\left(\pi + \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi - \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when H = 0 and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

<u>Examples:</u>

Second time at water level – first time at water level:

$$2\left(\pi + \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) = 7.02685... - 3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right)\right) = 2(5.35589... - 3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ **which is not immediately obvious but may be seen as an overall method.**

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

<u>Special Cases in (c):</u>

Note that if candidates have an incorrect α and have e.g. $3 + 2\sqrt{5}\cos(0.5t - 0.464)$, this has no impact on the final answer. So for candidates using $3 + 2\sqrt{5}\cos(0.5t \pm \alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

<u>Some values to look for:</u>

$$0.5t \pm "0.464" = \pm 2.306,\ \pm 3.977,\ \pm 8.598,\ \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the “3” then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.